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CRITIQUE OF ROLE OF TIME ALLOCATION IN RIVER BASIN MODEL

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by

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Grant No. R-801521
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SECTION I

INTRODUCTION

The River Basin Model is a computer simulation model which focuses on man - environment interactions and mutual interdependencies. As such it must model the internal behavior of both natural and man-made systems as well as identify and replicate the linkages between them. At the core of the human sphere is the household, as represented in the River Basin Model through the population units of the Social Sector. Households form the core not so much because they initiate action - indeed, they often tend to follow the lead of other sectors or institutions - but rather because their well-being, now and in the future, is, or should be, the ultimate objective of the operations of man-made systems. Dependent as they are on the state of the natural environment for so much of this well-being, households and the institutions designed to serve them must recognize the delicate relationships between their activities and the state of that environment. The River Basin Model has been developed to simulate those relationships in order to promote that recognition.

The River Basin Model takes the differential use of time in activities as the essence of household behavior, and quite rightly so, for it is in connection with such allocation of time that households make demands of the other sectors as citizens and consumers and participate in their operations as directors and employees. Associated with the devotion of household time to activities are flows of energy from participants through the physical instruments they manipulate and the resultant movement of people, goods and money through the system to assure that the proper actors and instruments are in the right place at the right time for activity performances. Not only are these exchanges and movements related to the ultimate time-use of goods in activities, but also they are themselves time-consuming processes necessary for overcoming the separation of spatially as well as temporally extensive activities. Various intensities of human activities over durations make simultaneous demands of the natural environment, altering its structure through the use and pollution of its resources.

The concern of this critique is the appropriateness of this view of time use as the crux of household behavior and as the essential link between that behavior and the operations of other institutions in the human system. From the remarks above it should be obvious that this review is begun with a bias favorable to the usefulness of the household time allocation perspective. It is hoped that in the course of the critique the arguments for the perspective will become apparent, along with a recognition of the problems which do arise in its application. However, no attempt will be made here to present a formal argument as to the general validity of this perspective, although much attention will be given to micro-economic consumer theory with time use.

This critique, then, will evaluate the use of the household time allocation mechanism in the River Basin Model for describing the behaviors of households and relating them to the behaviors of other sectors. Most of the attention will go to detailed examinations of the time and location assignment procedures used in the model, and therefore the critique tends to be short-sighted. That is, it does not attempt any new model-building efforts. Instead, it characterizes and criticizes the present structure of the model and offers suggestions for its improvement in terms of comprehensiveness and behavioral realism. Finally, although it is not responsible for development of a new simulation procedure to replace the gaming format now used, the critique will show how the consistancy of the present assignment structure with micro-economic theory (given certain modifications) suggests using the static consumer optimizing model in that dynamic simulation.

The critique is divided into two parts. The first is a review of the literature on utility theory for consumer behavior in which time is a dimension (stock). This review is condensed from the author's doctoral dissertation (forthcoming) and will serve both as a frame of reference for evaluating Social Sector manipulations in the River Basin Model and as a source of suggestions for the development of the simulation procedure for population unit decision-making. The second part consists of that evaluatioh of the model and ends with some thoughts on that simulation. It had been hoped that some simple numerical sensitivity tests of certain parts of the model might be undertaken, but this did not prove feasible.

The use of the time allocation, or time budget, technique was found to be innovative and appropriate for the tasks demanded of it in the model. There are many ways in which it could be tightened up to close gaps this reviewer thinks exist in its structure at present, but most of these would require only minimal effort. Much research does need to be done to check out the reasonableness of the parameter values assumed in the model. At this time there is no published information adequate for this prupose, but the model might be fit to data such as that for the Washington study referred to in this critique. The Quality of Life Index seems to lack the comprehensiveness over choice variables required of a utility function, and its form should be reconsidered in the development of the procedure for simulating time allocations. These are technical matters, however, and do not qualify the general applicability of the time use mechanism for population units.

PART ONE

REVIEW OF THE THEORY OF TIME USE

SECTION II

CLASSICAL THEORY AND THE RIVER BASIN MODEL

The view of household behavior presented in the River Basin Model is similar to that of classical microeconomic theory of consumer behavior. In both the consumer has scarce resources to be allocated to various consumption activities, and it is presumed in both that it is possible to assign a numerical value to any particular allocation of resources to consumption to represent the (relative) "utility" or satisfaction arising from that allocation. In the classical theory the perspective is a static one. The consumer makes his allocational choice so as to maximize his utility once and for all under conditions of perfect knowledge of the state of the environment and of the outcomes of his choices. As a simulator of systems' behavior over time, the River Basin Model takes a dynamic perspective, with households making repeated allocations in a changing environment about which they have only limited knowledge. However, in its present form the River Basin Model incorporates constructs from the static classical theory. Therefore, it is felt that a proper critique of the model should begin with a thorough review of those areas of and extensions to classical theory which are relevant to the model. Moreover, in moving from a gaming approach to internal machine simulation of household behavior, the classical theory and its extensions should supply hints for the construction of appropriate mechanisms for determining revised consumer resource allocations in a changing environment. For these reasons, and to provide a general frame of reference in discussing consumer behavior, this chapter and several which follow will be devoted to the presentation of static optimizing models of consumer behavior in the classical vein.

Classical and Modern Utility Theory of Consumer Behavior

Central to the utility theory of consumer behavior is the "utility function" or "preference function" which assigns relative weights or "preferences" to individual components of consumption and produces a single value as the utility of that set of components, that set representing as it does an allocation of scarce resources to different consumption activities. In the classical theory a consumption activity was simply the purchase of a certain amount of a particular good. Later extensions to the theory consider use of goods explicitly as well as their purchase. The amounts of the goods purchased were the arguments to the utility function. In the classical theory there was generally one scarce resource, money (or income), to be allocated to the purchases of goods in various amounts. The problem for the consumer was to spend his income in such a way that his utility was maximized. In mathematical form the problem was to

Maximize $U = \mathcal{U}(x_i)$ (objective function)

Subject to $Y - \sum_{i=1}^{n} p_i x_i = 0$ (income constraint)

where U is the utility level

$\mathcal{U}(x_i)$ is the utility function defined over the x_i

x_i is the amount of the i^{th} good purchased, $i = 1, \ldots n$

p_i is the price of the i^{th} good

Y is the amount of income to be spent

As stated here, the problem assumes that all income will be spent. Normally it is assumed that the marginal utility of a good is positive

$$\frac{\partial \mathcal{U}}{\partial x_i} > 0$$

so that more of the good is preferred to less, and that utility increases at a decreasing rate ($\mathcal{U}$ is a single-valued, monotonic function),

$$\frac{\partial^2 \mathcal{U}}{\partial x_i^2} < 0.$$

With all income spent, maximizing U subject to the budget constraint is equivalent to maximizing the following Lagrangian form of the objective function:

$$L = U + \lambda(Y - \sum_i p_i x_i)$$

The maximal level of U is achieved by that allocation of income Y to goods x_i which renders the partial derivatives of L with respect to the x_i all equal to zero. These "first order conditions"[1] are

---

[1] Higher order conditions for a maximum will not be presented in these simple expositions of the theory and its extensions.

$$\frac{\partial L}{\partial x_i} = \frac{\partial \mathcal{U}}{\partial x_i} - \lambda p_i = 0 \qquad i = 1, \ldots, n$$

where λ is the Lagrangian multiplier or the "shadow price" on the income resource. These conditions, plus the budget constraint, produce a system of $n + 1$ equations which can be solved (if appropriate conditions are met) for the $n + 1$ unknowns x_i and λ. λ is called the shadow price on income (money) because it is the value of the change in utility at the optimum allocation of the x_i with a small change in income:

$$\begin{aligned} \frac{dU}{dY} &= \sum_i \frac{\partial \mathcal{U}}{\partial x_i} \frac{dx_i}{dY} \\ &= \lambda\Big(\sum_i p_i \frac{dx_i}{dY}\Big) = \lambda \end{aligned}$$

Modern versions of the classical theory consider U a relative index, with arbitrary origin and scale and specific to the individual. This view of $\mathcal{U}$ as an ordinal utility function precludes comparison of utility levels between individuals, in contrast to the assumption of cardinal utility of much classical theory which would allow such interpersonal comparisons. Modern theory also views the utility maximization problem as a general non-linear program in which there may be several resource constraints, a constraint may be slack (some resource not completely exhausted at the optimum), and a choice variable (such as the x_i) need not be undertaken at a positive level. For example, the consumption problem above might be generalized as

Maximize $U = \mathcal{U}(x_i)$ (objective function)

Subject to $x_i \geqslant 0$ $\quad i = 1, \ldots, n$ (non-negativity constraints)

$Y_k - \mathcal{P}_k(x_i) \geqslant 0$ $\quad k = 1, \ldots, m$ (budget constraints for resources k)

where the $\mathcal{P}_k(x_i)$ are functions of the choice variables x_i and may be thought of as uses of the resources Y_k. The Lagrangian form of the objective function is now

$$L = U + \sum_{k=1}^{m} \lambda_k (Y_k - \mathcal{P}_k(x_i))$$

and the first order conditions for the maximum are

$$\frac{\partial L}{\partial x_i} = \frac{\partial \mathcal{U}}{\partial x_i} - \sum_{k=1}^{m} \lambda_k \frac{\partial \mathcal{P}_k}{\partial x_i} \leqslant 0$$

$$\begin{cases} = 0 \text{ if } x_i \geqslant 0 \\ < 0 \text{ if } x_i = 0 \end{cases} \qquad i = 1, \ldots, n$$

$$\frac{\partial L}{\partial \lambda_k} = Y_k - \varphi_k(x_i) \geqslant 0$$

$$\begin{cases} = 0 \text{ if } \lambda_k \geqslant 0 \\ > 0 \text{ if } \lambda_k = 0 \end{cases} \qquad k = 1, \ldots, m$$

The last m conditions are merely the budget constraints, which we note will have multipliers or shadow prices which vanish when there are unused resources.

There are two concepts in this model of consumer behavior which should be identified here. The first is the "marginal rate of substitution" of an argument, say x_j, for another, x_i, at the optimum. This is given by the slope of the indifference curve or utility contour in their direction at that point, and is equal to the negative ratio of their marginal utilities:

$$\frac{dx_i}{dx_j} = \text{MRS (j for i)} = - \frac{\partial \mathcal{U}/\partial x_j}{\partial \mathcal{U}/\partial x_i}$$

For "normal" arguments with positive marginal utility, this slope is negative in keeping with the concept of "substitution"; a positive slope indicates complementarity. From the first order conditions, this

$$= \frac{\sum_{k}^{m} \lambda_k (\partial \varphi_k / \partial x_j)}{\sum_{k} \lambda_k (\partial \varphi_k / \partial x_i)}$$

In the classical case, with only one constraint, the negative of the marginal rate of substitution between two goods i and j is simply the ratio of their prices.[2] In the general multi-constraint problem, marginal rates of substitution involve shadow prices on resources as explicit weights.

This brings up the second concept, which is the relative shadow price of one resource in terms of the shadow price of another resource at the optimum. Such a relative shadow price of resource k in terms of resource k' is given by the ratio

$$\frac{\lambda_k}{\lambda_{k'}}$$

---

[2] There, $\varphi(x_i) = \sum_{i}^{n} p_i x_i$

and $\frac{\partial \varphi}{\partial x_i} = p_i$, etc.

which we know to be equal to the ratio of the marginal utilities of external marginal changes in the levels of the stocks

$$\frac{dU/dY_k}{dU/dY_{k'}}$$

This ratio gives in effect the negative of the marginal rate of substitution between the stocks, or the amount of one stock which is a utility-equivalent of a unit of the other at the margin. One point should be stressed, and that is that under the ordinal utility interpretation shadow prices for resources are in utility units per unit of resource. Therefore, their absolute magnitudes are arbitrary. Thus it is precisely the ratio of shadow prices which is of significance and forms a metric for making interpersonal comparisons.

Theoretical Constructs in the River Basin Model

Households[3] in the social sector of the River Basin Model are faced with the allocation of two scarce resources (although, as we will see, these are highly interdependent allocations), these resources being time and money. The model introduces as a choice parameter a variable called the "dollar value of time in travel" which proports to measure the marginal rate of substitution of money (in travel costs) for travel time in route and modal selection for the journey to work. This selection can influence in turn job assignment and residual time for use in other activities, and, of course, both gross and net income for use in the purchase of goods for activities. Thus this substitution should also reflect more general tradeoffs between leisure time and money for expenditures.

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[3] Actually, groups of households as "population units." See the discussion to follow.

Classical and modern utility theory give specific relationships for such parameters, which are, after all, ratios of shadow prices and marginal utilities. Although the gaming version of the River Basin Model does not force any particular choice of values for such parameters, a simulation version of the social sector in the model could incorporate choice of parameter values which are at least explicitly suboptimal and deviate from optimal values in a systematic fashion, if indeed they are not themselves optimal values for a static perfect knowledge model.

However, even in the static perfect knowledge case there is considerable variation in optimal parameter values according to the way in which consumer behavior is visualized and functional relationships identified. For this reason a good deal of effort will be given over in this report to demonstrate the types of shadow price relationships between time and money which would develop under different specifications of consumer behavior and satisfaction. In concluding sections, recommendations will be made for the application of the theory and its constructs to a dynamic simulation of the sector.

Individual Versus Group Behavior and Preferences

Behaving units in the social sector of the River Basin Model are "population units" or groups of households with specific demographic structures, whereas micro-economic utility theory of consumer behavior refers to the individual, or at most the household, as the behaving unit. However, treatment of population units in the River Basin Model seems for all practical purposes independent of actual population unit definition and thus susceptible to arbitrary scaling of population unit size without appreciable effect on the functioning of the model, subject to scale adjustments in other sectors.[4]

In extrapolating from micro-economic theory of the individual consumer to the behavior of groups of consumers, then, we face the "aggregation problem" in developing preference functions and behavior dimensions for groups as collections of individuals. On the other hand, it is frequently remarked that the attempt to model individual consumer behavior will be as futile as the attempt to model the behavior of individual atomic particles has been and that understanding is best achieved by viewing the behavior of groups of systems of individual units as a whole. However, aggregation of postulates for individual behavior to those for behavior of groups as collections of individuals, however this aggregation is accomplished, cannot be expected alone to provide a view of the "whole" which is fundamentally different from - and free of the defects of - the view of the parts separately. Therefore, a "solution" to the aggregation problem for group preferences would not in itself resolve disputes about the reasonableness of postulating the existence of preference functions for individuals.

---

[4] This should become apparent in the expositions of Part Two.

The value of analyzing collective rather than individual behavior lies in the possibility of finding simplifications which might present themselves as "central tendencies" for the aggregate. In the River Basin Model, population unit behavior is no more easily analyzed than would be the behavior of a single household, were it not for the limited capacity of the computer in handling a large number of behaving units. Aggregation, then, appears to have been a practical rather than behavioral matter, and efficiency or model feasibility are certainly justifiable reasons for aggregation. Preference analysis has much to offer in its own right, in terms of its ability to provide constructs and indices the values of which can be of considerable interest to planners despite their limitations.

It is our view here, then, that the population units of the social sector of the River Basin Model are in fact behaving units indistinguishable from "individual consumers" and that the "aggregative" preference function approach of the model can be reviewed constructively by reference to and comparison with classical theory for individuals and its recent extensions. If this type of structure - treating behaving units deterministically - is thought to be inappropriate for machine simulation of the sector, then some fundamentally different approach from preference analysis should be pursued, such as a truly probabilistic structure.[5] Development of such a model is not the charge here, only the critique of the present structure. As stated above, suggestions for deterministic machine simulation based on the present preference function approach will be offered in conclusion to this critique of the River Basin Model.

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[5] Since consumer demands are not automatically met in the model, outcomes to consumers are not strictly deterministic. However, machine simulation with preference functions would most probably involve deterministic derivation of those demands themselves each time period. A "truly probabilistic structure" would not only determine outcomes stochastically, as is the case to some extent in the model as it stands, but would also produce probabilistic demands on the part of the consumers (population units) and thus yield probability distributions for outsomes and the state of the system as a whole.

SECTION III

THE "DEMAND FOR LEISURE" MODEL AND ITS ASPATIAL EXTENSIONS

Among models for simulating the urban system the River Basin Model is unique in its placing of time on an equal footing with money in resource allocational problems facing the consumer, or population units in the Social Sector. Indeed, time allocation even takes precedent over monetary allocation in the model's view of consumer work, social, and recreational behavior. The special contribution that time budget analysis can make to the study of pollution and environmental systems in the urban area is the translation of institutional ties between the Social and Economic sectors (as measured by money flows) into extensive (time) measures of consumer demand for environmental qualities in various use intensities (activities) leading to subsequent determination of the polluting and quality-reducing implications of those demands.

It is the objective of this and the chapters immediately following to examine the place that time allocation and the detailing of consumer (household) activities has recently found in the micro-economic theory of consumer behavior, where once the only "activity" implied was the instantaneous acquisition and consumption of physical quantities. This chapter will introduce the time budget and the analysis of different household activities in the utility theory of consumption in an aspatial context. The next chapter will consider trip-making behavior, and the following one will bring up residential location and space use with a time budget perspective. In each case we will be especially interested in the marginal rates of substitution between variables and in the shadow prices on time and money, since these constructs have relevance for the operation of the River Basin Model.

The Classical Demand for Leisure and Supply of Labor Model

One of the simplest models in the theory of consumer behavior, and historically the first to use time as a static dimension (a stock) in that behavior, is the theory of the demand for leisure, or conversely, the theory of the supply of labor offered by the worker. This theory ties together consumer demand for goods for consumption and worker participation in the labor market, selling his labor, as measured in time units.

The individual attempts to find the allocation of time to work and leisure which maximizes his utility as a function of income earned (goods consumed) and time spent for leisure or work at a given wage rate per hour. This is a two constraint, or two budget (time and money), model. When the wage rate is varied, the trace of the amount of labor the consumer-worker will offer to work, as a function of that wage rate, is called the individual's "labor offer curve." This curve is used in the theory of wages for aggregation into market functions for the supply of labor available to firms at the wages they offer.[1]

Leisure itself is treated somewhat ambiguously in this model. Some writers consider leisure time a proxy for personal inputs of effort or energy in consumption as the use of goods, such inputs requiring time for their implementation. Other theorists view leisure as an alternative source of utility distinct from the utility of income as a proxy for consumption.

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[1] Most texts of the theory of wages describe the work-leisure model and market relationships in some detail. See, for example, Richard Perlman, Labor Theory, (New York: John Wiley), 1969.

In the first case we have in effect time and goods as inputs to a production function for consumption, with utility defined over the resulting level of consumption. In the second case leisure time and income are direct sources of utility with no reference to consumption as an explicit activity. These divergent interpretations lead to virtually the same specification of the model in general terms, however. In either case, work time could be an argument to the utility function in addition to income or consumption (goods and leisure), but it is usually omitted from that function. That simplification has important implications, and the fact that widely differing results can be obtained under treatments of work time is often overlooked in applications of this model.

Under the production function approach to consumption, the work-leisure problem for the consumer may be written as follows:

Maximize $U = \mathcal{U}(t_w, t_1, x)$

Subject to $t_w, t_1, x \geq 0$

$wt_w - x \geq 0$ money budget

$T - t_w - t_1 \geq 0$ time budget

where

$x = \sum_{i=1}^{n} p_i x_i$	is gross consumption expenditures, the sum of goods purchased times their prices.
t_w	is work time
w	is the wage rate
wt_w	is the income earned
T	is the total stock of time (length of the time period)

In this statement of the problem, there can be less consumption expenditure than income earned. In the alternative leisure time interpretation, total income Y would replace expenditures x in the utility function and in the money budget, and the money budget constraint would become a definitional equation. Whether or not this change in interpretation would imply a different type of utility function, in terms of its properties, is a matter for further speculation.[2] The non-linear programming problem then could be simplified by substitution to eliminate the money budget equation and either the income or work time variable.

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[2] For example, in trading off the utility of income against the utility of leisure time with no reference to consumption activities per se, there may seem to be no reason to expect satiation effects to ever set in. But with consumption as a function of leisure time and goods inputs, it is easy to see how satiation points could be reached at which there is so little time for consumption relative to the amounts of goods to be consumed that no further increase in utility with an increase in goods can occur without a concurrent increase in leisure time. The phenomenon of "conspicuous consumption" of additional goods without "using" them could postpone or eliminate satiation, of course. The point is that the outcome of the model can vary considerably under these different specifications.

In the problem as stated above, the first-order conditions for a maximum are

$$\frac{\partial \mathcal{U}}{\partial t_w} + \lambda W - \theta \leq 0 \qquad \begin{cases} = 0 \text{ if } t_w \geq 0 \\ < 0 \text{ if } t_w = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_l} - \theta \leq 0 \qquad \begin{cases} = 0 \text{ if } t_l \geq 0 \\ < 0 \text{ if } t_l = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial x} - \lambda \leq 0 \qquad \begin{cases} = 0 \text{ if } x \geq 0 \\ < 0 \text{ if } x = 0 \end{cases}$$

$$x - Wt_w \leq 0 \qquad \begin{cases} = 0 \text{ if } \lambda \geq 0 \\ < 0 \text{ if } \lambda = 0 \end{cases}$$

$$t_w + t_l - T \leq 0 \qquad \begin{cases} = 0 \text{ if } \theta \geq 0 \\ < 0 \text{ if } \theta = 0 \end{cases}$$

The parameters λ and θ are the shadow prices associated with the money and time budget constraints, respectively. Each will be positive if its constraint is binding (an equality at optimal allocation of resources). As such, they represent the marginal utilities of money and time at the optimum, being the increases in utility for slight "relaxations" of the constraints (through additions to stocks):

$$\frac{dU}{dT} = \frac{\partial \mathcal{U}}{\partial x}\frac{dx}{dT} + \frac{\partial \mathcal{U}}{\partial t_w}\frac{dt_w}{dT} + \frac{\partial \mathcal{U}}{\partial t_1}\frac{dt_1}{dT} = \theta$$

$$\frac{dU}{dS} = \frac{\partial \mathcal{U}}{\partial x}\frac{dx}{dS} + \frac{\partial \mathcal{U}}{\partial t_w}\frac{dt_w}{dS} + \frac{\partial \mathcal{U}}{\partial t_1}\frac{dt_1}{dS} = \lambda$$

where S is the amount by which consumption expenditures exceed earned income.[3] These shadow prices are also often referred to as the marginal values of time and money, the unit of value (utility) being implicit.

The ratio of θ to λ is the ratio of marginal utility per unit time to marginal utility per dollar, in terms of stocks. As such it is often referred to as the "dollar value of time." In general, this valuation need not refer to specific uses of time or dollars, although its value depends on those uses. In this model, if at the optimum there is not satiation in leisure or consumption expenditures, so that these arguments, and the partials of the utility function with respect to them are positive, then this dollar value of time equals the marginal rate of substitution of leisure time for consumption expenditures[4]

---

[3] Alternatively, S is a stock of other non-wage income. In the statement here S is assumed to be zero, but its inclusion in the money budget at a positive level would not alter the first order conditions.

$$\frac{\theta}{\lambda} = \frac{dU/dT}{dU/dS} = \frac{\partial \mathcal{U}/\partial t_l}{\partial \mathcal{U}/\partial x}$$

and represents the full opportunity cost of spending a unit of time in leisure rather than in non-leisure (income production for the purchase of goods). This, then, is here the (marginal) value of that leisure time, and from the first order condition for work it may also be written

$$\frac{\theta}{\lambda} = W + \frac{\partial \mathcal{U}}{\partial t_w}$$

If work time is desirable (positive marginal utility), then this opportunity cost of leisure and thus its value is greater than the wage rate, while it is less if work time is undesirable at the optimum (negative marginal utility). The traditional theory does not define utility over work, so that that partial derivative vanishes and the marginal unit of leisure is valued at the market price for labor time, the wage rate. This familiar - but very special - result is widely applied as a rule of thumb in valuing time but does not appear to have much behavioral justification. Introduction of other activities, such as travel, qualifies that result even further, as we shall see.

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[4] Strictly speaking, the negative of that marginal rate of substitution.

The labor offer curve itself may be obtained for t_w as a function of w by substituting for the shadow prices and w in the first order condition for t_w:

$$t_w = \frac{x(\partial\mathcal{U}/\partial x)}{\partial\mathcal{U}/\partial t_1 - \partial\mathcal{U}/\partial t_w} = g(w)$$

This can be denoted as some function g of w, since at the optimum the other arguments and marginal utilities may be solved for as functions of w and other constant parameters. Since

$$\frac{dU}{dw} = \frac{\partial\mathcal{U}}{\partial x}\frac{dx}{dw} + \frac{\partial\mathcal{U}}{\partial t_w}\frac{dt_w}{dw} + \frac{\partial\mathcal{U}}{\partial t_1}\frac{dt_1}{dw} = \lambda t_w \geq 0 \quad ,$$

a higher wage is always preferred by the consumer-worker to a lower one. Whether or not he will increase his labor offer with an increase in the wage at a particular level of that wage depends on the nature of g, which in turn depends on the nature of the utility function. Traditional theory hypothesized a "backward bending" curve, where t_w increased with w up to a point, then decreased with further wage increases, although not enough to ever cause a decrease in consumption expenditures.

Consumption Activities in the Labor-Leisure Model

A straightforward modification of the demand for leisure model is the disaggregation of gross expenditures x into individual physical amounts for each good, x_i, $i = 1, \ldots, n$. When x is replaced by the x_i separately in the utility function and by its algebraic equivalent, $\sum p_i x_i$, in the money budget constraint, where p_i is the price of good i, then the first order conditions for the amounts purchased are

$$\frac{\partial \mathcal{U}}{\partial x_i} - \lambda p_i \leq 0 \quad \begin{cases} = 0 \text{ if } x_i \geq 0 \\ < 0 \text{ if } x_i = 0 \end{cases}$$

These conditions are no more than the first-order conditions of the classical problem without time, which now may be seen to fit into the demand for leisure model with no obvious inconsistencies.

The next simple extension would be to identify particular activities within the leisure time component and to make specific the amount of time, t_k, and of each good i, x_{ik}, used in each activity $k = 1, \ldots, m$. Suppose that the utility of a good can vary by its use, so that the x_{ik} are used separately in the utility function in place of the gross amounts $x_i = \sum_k x_{ik}$.[5] With the individual times t_k also in the utility function in place of t_l, the first-order conditions for these arguments become $(k = 1, \ldots, m)$

$$\frac{\partial \mathcal{U}}{\partial x_{ik}} - \lambda p_i \leq 0 \qquad \begin{cases} = 0 \text{ if } x_{ik} \geq 0 \\ < 0 \text{ if } x_{ik} = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_k} - \theta \leq 0 \qquad \begin{cases} = 0 \text{ if } t_k \geq 0 \\ < 0 \text{ if } t_k = 0 \end{cases}$$

This means that marginal utilities of time in all activities (non-work) undertaken must be equal, that the ratios of the marginal utilities of goods however used are still equal to the ratios of their prices, and the ratio of the marginal utilities of an activity time to a good input to that activity is equal to the ratio of the dollar value of time (as the price of time) to the price of the good.[6]

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[5] Again, this treatment may be interpreted as subsuming in the utility function production functions for each activity in time and goods, with utility defined over the "levels" of the activity as given by those production functions. That interpretation, however, implies separability properties which are not prerequisite to the theory.

A simplification to this type of extension to the leisure-labor model that was made first by Becker[7] and, later by DeVany,[8] is the assumption of fixed proportions in mixes of goods in activities. What is done is to define each of a set of consumption activities in terms of an identifiable "composite commodity" unique to that activity.[9] An activity becomes by definition the "consumption" of the appropriate composite good. This one-to-one correspondence between activities and mixes of goods avoids confusion which might arise in the variable proportions model above concerning the identifiability of activities having, at the optimum, similar input structures. On the other hand, to remove input substitution within activities seems to overly restrict the range of choices open to the consumer.

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[6] For examination of higher-order conditions in simple cases similar to these, see the Mathematical Appendix to Staffan B. Linder, The Harried Leisure Class, (New York: Columbia University Press), 1970.

[7] Gary S. Becker, "A Theory of the Allocation of Time," Economic Journal, Vol. 75, September 1965, pp. 495-517.

[8] Arthur DeVany, "Time in the Theory of the Consumer," Professional Paper No. 36, Center for Naval Analyses, Arlington, Virginia, June 1970.

[9] This usage of the term "composite commodity" differs from that in comparative statics, where a set of goods can be treated as a composite commodity if their prices change proportionately.

If the amount of the composite good k consumed in its activity is denoted A_k, then the requirement of proportional inputs for good i can be written as

$$x_{ik} \equiv c_{ik}A_k$$

The scale of the composite goods is arbitrary, thus so is the scale of the c coefficients.

A related simplification made by both Becker and DeVany is the assumption that the consumption of each "unit" of the composite good k takes a fixed amount of time, b_k. Thus

$$t_k \equiv b_kA_k$$

Again, the scale of the b_k is arbitrary. Indeed, any one good or time could have been picked as "numeraire" and used in place of the composite goods, which are, in effect, indexes for the levels of the consumption activities.[10] These proportionality relations may be used to restate the budget constraints in terms of the A_k by substitution. With utility defined over these activity (or composite good) levels, first order conditions for a maximum become

$$\frac{\partial \mathcal{U}}{\partial A_k} - \lambda\left(\sum_i p_i c_{ik}\right) - \theta b_k \leq 0 \quad \begin{cases} = 0 \text{ if } A_k \geq 0 \\ < 0 \text{ if } A_k = 0 \end{cases}$$

The term in parentheses is the price P_k of the composite good (activity), being the sum of the prices of component goods weighted by their input coefficients. For two composite goods k and m

$$\frac{\partial U/\partial A_k}{\partial U/\partial A_m} = \frac{P_k + (\theta/\lambda)b_k}{P_m + (\theta/\lambda)b_m}$$

where P_k and P_m are the composite prices. The expression

$$\pi_k = P_k + (\theta/\lambda)b_k$$

is often called the "full price" or the "shadow price" of composite good (activity) k. In this price, consumption time per unit of k is valued at the dollar value of time, (θ/λ), and the resulting value is added to the composite market price P_k for the good to give its full or shadow price in terms of both of the consumer's scarce resources, time and money.

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[10] DeVany does, in fact, use time in this manner.

Work may be included among these activities, or the stock of income may be taken as given. In the latter case, there is a good chance that one constraint is slack. Indeed, with fixed proportions this is more than likely, meaning that one shadow price would be zero and that all shadow prices and one set of real prices, either the P_k or the b_k, would be eliminated from the marginal utility ratios above. With income generation within the model, however, both constraints are likely to be binding. Under the traditional assumption of no (dis-) utility to work, the dollar value of time is then

$$\frac{\theta}{\lambda} = \frac{W - \sum_i p_i c_{iw}}{b_w}$$

There the value of time in dollars is the wage rate corrected for the costs of inputs to work supplied by the consumer-worker himself. With no such inputs of physical goods ($c_{iw} = 0$, all i), b_w can be set to unity and all other b_k's scaled according, since that scale is arbitrary, yielding the familiar $\theta/\lambda = W$. In any case, the full prices π_k are then constants in the system, and the structure of the Becker-DeVany activity analysis in A_k and π_k is strictly analogous to traditional non-time analysis in physical quantities x_i and their prices p_i.

The chain rule for partial derivatives may be used to restate the first order conditions in terms of the original arguments x_{ik} and t_k for comparison with the variable proportions case presented previously.[11]

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[11] In the variable proportions case, the production function interpretation implies relations such as

$$A_k = \mathcal{a}_k(t_k, x_{ik})$$

with the $\mathcal{a}_k$ as the production function for activity k in terms of time and goods inputs, in variable proportions. When utility is defined over the A_k, simple substitution of the production functions into the utility function yields a (separable) utility function in t_k and x_{ik}, as used previously.

$$\frac{\partial \mathcal{U}}{\partial x_{ik}} \equiv \frac{\partial \mathcal{U}}{\partial A_k} \frac{\partial A_k}{\partial x_{ik}} = \frac{1}{c_{ik}} \frac{\partial \mathcal{U}}{\partial A_k}$$

$$\frac{\partial \mathcal{U}}{\partial t_k} \equiv \frac{\partial \mathcal{U}}{\partial A_k} \frac{\partial A_k}{\partial t_k} = \frac{1}{b_k} \frac{\partial \mathcal{U}}{\partial A_k}$$

Now the ratio of the marginal utilities for two goods used in the *same* activity is equal to the inverse ratio of their input coefficients, rather than equal to the ratio of the prices, as in the variable proportions case. Ratios of marginal utilities for arguments in different activities will involve ratios of the activities' full prices divided by the appropriate input coefficients as weights.

The assumption of fixed proportionality seems unnecessary in these theoretical models, although it does show under what circumstances the time and money model is analogous to traditional non-time analysis, and it may offer computational simplicities for empirical application. Judging from the substantial variation in times spent on activities by consumers surveyed in time budget studies,[12] it seems premature to posit strict proportionality of inputs, or at least to interpret such proportionality as any more than an average tendency. It may be that input substitution in activity performance needs to be constrained. However, the problem can be handled by postulating production functions or utility functions exhibiting diseconomies or satiation for arguments which are too high relative to the levels of other inputs to their activities.[13] Theorists do not like to consider satiation, as its existence can complicate the conditions for an optimum. From a behavioral viewpoint, however, it is a likely phenomenon, certainly at the level of the individual consumer in the short run. This point will come up again in considering activity frequency as well as duration.

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[12] See, for example, Philip G. Hammer, Jr., and F. Stuart Chapin, Jr., *Human Time Allocation: A Case Study of Washington, D.C.*, (Chapel Hill: Center for Urban and Regional Studies, University of North Carlina), March 1972.

Consumption Technology as Universal

Activity definition per se is an extremely difficult problem, conceptually and empirically, for there may be cognitive distinctions not reflected by differences in input allocations to activities. Becker's approach has been to posit technological relationships among inputs to activities as if these were independent of consumer preferences for activities.[14] The obvious advantage of this approach is that activity definition becomes universal even if preference is peculiar to the individual. Kelvin Lancaster[15] has gone one step further and suggested that the experiencing of activity performance is multi-dimensional and that the intensity of the activity on each dimension contributes to the utility of the activity. Preferences, then, are directly for types of stimulii as represented by those dimensions, and only indirectly for activities in terms of their ability to impart those stimulii.

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[13] Indeed, DeVany specifically challenges Becker's assertion that proportionality relationships constitute proper production functions for activities. His solution is to posit one-to-one correspondence between goods and activities, which seems hardly more than a technical sleight-of-hand.

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[14] Another input to activities which has been suggested is "energy" or "effort", although identification of the appropriate stock level and use coefficients, and even "production" of energy in other activities, might be difficult and quite particularistic. See Mark Menchik's unpublished paper, "Towards a Generalization of the Theory of Consumer Behavior," (Philadelphia: Regional Science Research Institute), April 27, 1967.

[15] Kelvin Lancaster, "A New Approach to Consumer Theory," Journal of Political Economy, Vol. 74, 1966, pp. 132-157.

In Lancaster's scheme, then, an activity is characterized not only by its input structure, as with Becker, but also by its output structure in terms of the stimulii it imparts. Lancaster assumes that the intensity level Z_m of stimulus (or "characteristic") m is a linear function of the activity levels A_k:

$$Z_m \equiv \sum_k e_{mk} A_k \qquad m = 1, \ldots$$

Utility, in turn, is defined over the stimulus intensities Z_m rather than over the activity arguments. He follows Becker in assuming proportional goods inputs to activities —

$$x_{ik} \equiv C_{ik} A_k$$

— and assumes that the coefficients e_{mk} as well as the C_{ik} are universal parameters independent of individual preferences for characteristics.

The optimization problem for the consumer becomes the problem of choosing levels for activities so that utility over characteristics is maximized, subject to the money budget constraint and the technological consumption relations above. With more activities to choose from than there are characteristics and with the possibility of experiencing the same characteristics in many different activities at varying intensities, something less than the full set of activities need be undertaken in maximizing utility even if utility is (as he presumes) a positive monotonic function of all characteristics. Because of the technical complexities thus created in the general case, optimization there is intractable analytically, although special cases can be handled analytically. It is therefore difficult to say much about the nature of optimal solutions. Lancaster does show how an "efficiency frontier" can be constructed to weed out activities that are strictly inferior to others in characteristics production under the money budget constraint and thus to identify among all possible activity combinations a much smaller subset which will contain the optimal solution. Choice among this reduced set along the efficiency frontier is called the "efficiency choice."

Time as an input to activities and a resource stock constraint is not included in Lancaster's scheme. Incorporation of time is possible, but the addition of the second budget constraint complicates the analysis because it further breaks up an already jagged efficiency frontier into additional segments for which only one constraint or the other is binding, with both binding only at corner points,if income is held constant. With variable income (i.e., work as an explicit activity), there can be segments on the frontier where both constraints are binding. These properties are demonstrated by an example worked out in the Appendix to this report, although they have not been demonstrated for the general case. One interesting property of the example chosen is that when both constraints are binding, the dollar value of time which results is independent of preferences and thus of the actual location of the optimal solution on that segment of the efficiency frontier where both constraints are binding. This is not expected to be a property of the general model, however.

Two Applications: Household Work and Overtime

There are numerous theoretical applications that can be made of the aspatial time and money model as it has been developed here. An example is the "home production" of consumer goods or services. This can be viewed as a problem in labor force participation[16] or as a problem in the maintenance of consumption goods for activities as an alternative to purchase of new goods.[17] Suppose that such a good or service x_s is available on the market at price p_s per unit, and that it may also be produced by the individual himself according to the production function $\mathcal{a}_s(x_{is}, t_s)$, with inputs of goods x_{is} and time t_s. The x_{is} are amounts of goods i bought on the market at price p_i, of course. Let the good or service, whether purchased or produced, be homogeneous in quality and used as an input x_{sk} to other activities k. Then the consumer optimization problem in variable proportions will have the additional constraint

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[16] In this problem, the question is whether a person's time is worth more on the labor market or in household tasks, as, for example, in the decision for potential secondary wage earners in a household to enter the labor market. One statement of this problem is given in O.H. Brownlee and Hohn A. Buttrick, Producer, Consumer, and Social Choice, (New York: McGraw-Hill), 1968, Chapter 13.

[17] Linder, op. cit., offers a different version of the discussion here, in which he treats "personal work" as a contributor to real income along with work for wages in the labor market.

$$a_s(x_{is}, t_s) + x_s - \sum_k x_{sk} \geq 0$$

specifying thst the uses of the good or service s in activities k not exceed supply via purchase (x_s) or production ($a_s(x_{is}, t_s)$).

When these new arguments - x_s, x_{is}, t_s, and x_{sk} - are introduced into the utility function and into the appropriate constraints, additional first order conditions for a maximum in the variable proportions model- are found as

$$\frac{\partial \mathcal{U}}{\partial x_s} - \lambda p_s + \phi \leq 0 \qquad \begin{cases} = 0 \text{ if } x_s \geq 0 \\ < 0 \text{ if } x_s = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial x_{sk}} - \phi \leq 0 \qquad \begin{cases} = 0 \text{ if } x_{sk} \geq 0 \\ < 0 \text{ if } x_{sk} = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial x_{is}} - \lambda p_i + \phi \frac{\partial a_s}{\partial x_{is}} \leq 0 \begin{cases} = 0 \text{ if } x_{is} \geq 0 \\ < 0 \text{ if } x_{is} = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_s} - \theta + \phi \frac{\partial a_s}{\partial t_s} \leq 0 \qquad \begin{cases} = 0 \text{ if } t_s \geq 0 \\ < 0 \text{ if } t_s = 0 \end{cases}$$

$$-x_s - a_s(x_{is}, t_s) + \sum_k x_{sk} \leq 0 \qquad \begin{cases} = 0 \text{ if } 0_s \geq 0 \\ < 0 \text{ if } 0_s = 0 \end{cases}$$

where $\phi \geq 0$ is the shadow price on the use-supply constraint for activity s. First order conditions for other arguments are the same as in the variable proportions model above. It is presumed that the partials of a_s with respect to t_s and the x_{is} are all non-negative.

Obviously, if the good or service s is not used in an activity k then the marginal utility of x_{sk} must be non-positive. If this were true for all activities, then ϕ would be zero and the good or service would be purchased or produced only if the appropriate arguments had sufficiently positive marginal utility, as for any other good or time input. Marginal utilities of x_{is} and t_s, like those for work inputs, need not be positive for the activity to be undertaken.

There are many different situations which can develop here. Let's take here only the simple case where the only input to home production of the service s is time, in terms of which the service is measured (as is x_s, but that is someone else's time). Then the first order conditions for the x_{is} drop out, and the condition for t_s becomes

$$\frac{\partial \mathcal{U}}{\partial t_s} - \theta + \phi \leq 0$$

If some $x_{sk} > 0$ at the optimum, then for both the home production and purchase on the market to occur the following relation must hold between the appropriate marginal utilities and the dollar value of time (obtained by substitution for ϕ):

$$\frac{\theta}{\lambda} = p_s + \frac{1}{\lambda}\left\{\frac{\partial \mathcal{U}}{\partial t_s} - \frac{\partial \mathcal{U}}{\partial x_s}\right\}$$

We note, too, that in this case ϕ is θ adjusted for the marginal utility of home production.

As a final, rather trivial example, suppose the consumer is neutral to home production and to purchase of the service, so that the marginal utilities of t_s and x_s vanish, then the first order condition for a maximum are

$$- p_s + \frac{\phi}{\lambda} \leq 0 \qquad \begin{cases} = 0 \text{ if } x_s \geq 0 \\ < 0 \text{ if } x_s = 0 \end{cases}$$

$$- \frac{\theta}{\lambda} + \frac{\phi}{\lambda} \leq 0 \qquad \begin{cases} = 0 \text{ if } t_s \geq 0 \\ < 0 \text{ if } t_s = 0 \end{cases}$$

which can hold simultaneously if

$$p_s = \frac{\theta}{\lambda} = W + \frac{\partial \mathcal{U}}{\partial t_w}$$

where the right hand side of this relation is the first order condition for work time. If the consumer is also neutral to work so that that marginal utility vanishes, then both conditions can hold only when $p_s = W$, assuming $\phi > 0$. If $W > p_s$, there will be only purchase and no self production. The opposite is true if $w < p_s$. The deciding factor is simply whether or not payment in kind (terms of p_s as the opportunity cost of home production) is greater than the wage rate so that home production returns more than work.

Again, there are other situations which can develop under other assumptions on the marginal utilities. Further, if there is a source of income other than work (e.g., a spouse's wages), there is the possibility that there will be no work undertaken if p_s is relatively high. This consideration is used in applying this model to the study of the labor force participation rate of married women faced with the choice between work and housework. In particular, labor offer curves can be derived to show at what wage level relative to p_s such a person would enter the labor market and what work time would be offered.

The consumer's ability to vary his work time at will has seemed to many theorists an unrealistic postulate of the labor-leisure model, and some attention has been given to models in which there was a fixed work shift, with the possibility of overtime work. The classic work is that of Moses ,[18] but a slightly different version will be developed here. Suppose that the worker must work a shift of length t_o at wage W per unit time but may also work overtime at the wage rate $\hat{W} = \alpha W$ (e.g., $\alpha = 1.5$ for "time and one-half"). Then if the total amount of time worked is t_w,[19] income earned is

$$Y = Wt_o + \hat{W}(t_w - t_o)$$
$$= W((1 - \alpha)t_o + \alpha t_w)$$

The constraint

$$t_w - t_o \geq 0$$

must be added here to assure that total work time not be less than the required shift. The first order condition for work time t_w becomes (e.g., in the variable proportions model)

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[18] Leon N. Moses, "Income, Leisure and Wage Pressure," Economic Journal, June 1962.

[19] This model presumes homogeneity of regular and overtime work, so that utility can be defined over the total as a single argument.

$$\frac{\partial \mathcal{U}}{\partial t_w} + \lambda \alpha w - \theta + \psi = 0$$

where $\psi \geq 0$ is the shadow price on the new work time constraint. This condition is an equation, since the problem requires $t_w > 0$.

If at the optimum $t_w > t_o$, then overtime is undertaken, and $\psi = 0$. Moses terms such a worker a "work preferer." Note that in such a case

$$\frac{\theta}{\lambda} = \alpha W + \frac{\partial \mathcal{U}}{\partial t_w} \frac{1}{\lambda}$$

so that the dollar value of time is based on αw rather than w alone. Here αw is the marginal wage rate even though all income is not earned at that rate. This valuation of time may differ from a specification of the model in which Wt_o was considered fixed income so that that portion of work time, t_o, would not figure into the utility function. Optimal t_w would also differ from that work time in the free-choice model at rate αw for all work time, since the bundle of goods obtainable for a given work time would differ, and that would affect time and money allocations.

If $t_w = t_o$ then $\psi > 0$, and we have a "leisure preferer" (Moses' term) whose shift forces him to work as much or more than he would choose otherwise. The role of this shadow price ψ is interesting. Suppose the work shift were lengthened without a change in the wage rate w. The resulting change in utility is

$$\frac{dU}{dt_o} = - \psi < 0$$

This change is negative, representing a loss of utility and a "tightening" of the work time constraint. However, if $\psi = 0$ and $t_w > t_o$, an increase in t_o will cause a change in the proportions of total work time t_w which are remunerated at rates w and αw. dt_w/d_{to} need not be unity, as it must be when $\psi > 0$. With $\psi = 0$,

$$\frac{dU}{d_{to}} = W(1 - \alpha)\lambda$$

This will be negative when $\alpha > 1$, since it would then mean a reduction in income for a given amount of work. Readjustment of t_w could be in either direction, as with wage changes in the simple labor-leisure model. A change in the wage rate w here could also cause a change in t_w in either direction. In general, then, ψ seems to represent a measure of the cost (in utility) of a forced suboptimal allocation of work time. ψ/λ would give a dollar equivalent of this cost.

Activity Frequency in Time and Money Allocations

A dimension which is rarely given comprehensive treatment in theories of consumer behavior is activity frequency, giving the number of activity occurrences or episodes to which time and money (goods) are allocated.[20] It seems intuitively obvious that individuals are not completely indifferent to the way a given amount of time in an activity is organized into discrete episodes. For example, the four day work week may not be preferred to the five, even for a given amount of work time and a fixed wage rate, if there is more dissatisfaction created with longer hours worked per day with less leisure time on those days than is gained through transportation savings (fewer work trips) and more continuous leisure time on the weekend.

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[20] Activity frequency has been studied in travel behavior, as described in the next chapter, but usually without reference to non-travel behavior. For further discussion and some empirical findings, see Hammer and Chapin, *op. cit*.

As is implied in this example, actual sequence of activities is a factor in the utility derived along with frequency-time-money relations. Sequencing, however, poses problems in optimization which are much too complex for the purposes here. It is felt, on the other hand, that activity frequency is one aspect of activity organization which can and should be represented in an optimizing model of consumer behavior. For transportation and public facility planning purposes there would seem to be a need to be able to explain traffic and facility use patterns in terms of underlying activity time, frequency and money tradeoffs. Such explanation would be crucial in relating changes in travel patterns to changes in facility use and to the organization of household activity in general. Again, to look at durations of individual episodes, as one aspect of sequencing, is to delve into more detail than can be expected to provide some simple insights into behavior. In addition, the optimizing models here must treat frequency as a continuous variable, allowing "fractional episodes." However, if frequency can be viewed as an average tendency from a sampling of time periods, this continuous feature of the variable should cause not more conceptual difficulty than it does in the case of physical goods, themselves more discrete than continuous. Finally, there is also the implication of homogeneity of activity episode or occurrence.

It is proposed, then, that the frequency f_k of an activity k be introduced to the utility function along with the time (t_k) and goods (x_{ik}) inputs. As with time and goods, it is expected that satiation will be a relevant property of utility relationships involving activity frequency. It should be easy to see, for example, how there could be points at which activity frequency is so great relative to the total time spent on the activity that the frequency could not be further increased without a loss in utility unless the total time spent on the activity were also increased (see footnote 2, this chapter). That is, for that frequency there is a finite "best" total time (or average time per occurrence), all else equal. Such satiation would be reflected in utility contours, or "indifference surfaces," which are concave to the origin in the vicinity of the origin, indicating substitution possibilities between arguments, but which turn away from the axes and the origin beyond the "satiation points," showing a shift to complementarity relations between arguments to the utility function.

Models with activity frequency explicit can vary greatly according to the types of activities considered. Occasionally a new constraint must be added, such as constraints which assure that for out-of-home activities there are enough trips to the activity location to permit the number of activity occurrences taking place there. These will be considered further in the next chapter. For out-of-home activities involving facility use there may be admission prices charged, so that frequency variables appear in the money budget. However, for activities at home and many "free" ones out-of-home, there will be no direct cost to frequency per se. This means that the first order conditions in such cases would simply be non-positivity constraints on the marginal utilities:

$$\frac{\partial \mathcal{U}}{\partial f_k} \leq 0 \qquad \begin{cases} = 0 \text{ if } f_k \geq 0 \\ < 0 \text{ if } f_k = 0 \end{cases}$$

Obviously, the marginal utility would be zero only at a satiation point for the frequency relative to the levels of other activities.

With an admission price charged for out-of-home activities, optimum frequency levels should be something less than the corresponding satiation points, since with such prices q_k, the first-order conditions

$$\frac{\partial}{\partial f_k} - \lambda q_k \leq 0 \qquad \begin{cases} = 0 \text{ if } f_k \geq 0 \\ < 0 \text{ if } f_k = 0 \end{cases}$$

demand positive marginal utility if the frequency is greater than zero. In connection with admission prices for out-of-home activities, it should be mentioned that there could be charges by the hour for facility use in place of or in addition to admission prices. This would place the time argument for the activity in both the time and money constraints, so that the first order condition for that argument would contain both shadow prices, as is the case with work.

In passing it should be mentioned that when more than one shadow price appears in first order conditions, it can be difficult to derive a system of equations in the unknown levels of the arguments from which those shadow prices have been eliminated by substitution. This can make analytical solution of the system for a specified utility function very complicated. Although this is a practical matter and does not qualify the theorems derived about behavior, it does explain why practical applications of the theory often go to extreme lengths to eliminate extra constraints and shadow prices from the consumer's optimization problem.

Since by definition the total time spent on an activity is the product of the frequency of occurrence and the average time per occurrence, substitution of that product for total activity time can put the problem in terms of those average times (denoted here $\bar{t}_k$) instead of the totals. This does not change the optimization problem, of course, but it may make manipulation of the first order conditions easier for theorem derivation. For example, the first order conditions for the classical labor-leisure model with activity frequency explicit become

$$\frac{\partial \mathcal{U}}{\partial x} - \lambda \le 0 \qquad \begin{cases} = 0 \text{ if } x \ge 0 \\ < 0 \text{ if } x = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_1} - \theta \bar{t}_1 \le 0 \qquad \begin{cases} = 0 \text{ if } f_1 \ge 0 \\ < 0 \text{ if } f_1 = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial \bar{t}_1} - \theta f_1 \le 0 \qquad \begin{cases} = 0 \text{ if } \bar{t}_1 \ge 0 \\ < 0 \text{ if } \bar{t}_1 = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_w} - \bar{t}_w(\theta - \lambda w) \le 0 \quad \begin{cases} = 0 \text{ if } f_w \ge 0 \\ < 0 \text{ if } f_w = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial \bar{t}_w} - f_w(\theta - \lambda w) \le 0 \quad \begin{cases} = 0 \text{ if } \bar{t}_w \ge 0 \\ < 0 \text{ if } \bar{t}_w = 0 \end{cases}$$

In this case, the leisure and work activities have marginal rates of substitution of frequency for average time inversely proportional to those arguments

$$\mathrm{MRS}(f_k \text{ for } t_k) = \frac{\partial \mathcal{U}/\partial f_k}{\partial \mathcal{U}/\partial \bar{t}_k} = \frac{\bar{t}_k}{f_k} \qquad k = 1,\ w$$

There are also explicit statements for the dollar value of time here in terms of the marginal utilities of frequencies and marginal rates of substitution for consumption expenditures:

$$\frac{\theta}{\lambda} = \frac{1}{t_1}\,\frac{\partial \mathcal{U}/\partial f_1}{\partial \mathcal{U}/\partial x} = \frac{1}{\lambda \bar{t}_w}\,\frac{\partial \mathcal{U}}{\partial f_w} + W$$

Again, if there is indifference to work time (average) and frequency, the dollar value of time equals the wage rate.

This has been only a brief introduction to the role of activity frequency in optimizing models of individual consumer behavior. More examples will be presented in the following chapters. No attempt will be made to cover systematically the wide range of model specifications that can arise with activity frequency, however. To delve into the implications of alternative frequency/time pricing arrangements for the use of facilities is of considerable interest in its own right, but it would contribute little to our critique of the River Basin Model. The present discussion, on the other hand, should provide a broad frame of reference for discussing that model.

SECTION IV

TIME AND FREQUENCY IN TRAVEL BEHAVIOR

Time is of special relevance in considering the spatial behavior of consumers because in many instances time may become a more restrictive factor in movement over space than is money. "Space" is used in a very general and often artificial sense in these applica--tions. It is standard procedure to view space as a "featureless plain" permitting movement in any direction at rates which are independent of direction and place per se. The alternative to this procedure is to allow movement only over given paths with travel times and costs specific to the links taken between nodes in the path. Although appropriate for simulation and operational modeling, this network approach is usually not helpful in deriving theorems from a classical type model of consumer behavior.

Straight-line movement between points is the first case of time use with spatial implications that we will consider. Such movement is presumed to occur because of the spatial separation of opportunities for different types of activities. The second case to be taken up (in the next chapter) is the actual use of space itself by the consumer as, for example, an input in its own right to activities requiring spatial extent, plus the choice of location(s) for assembling those inputs (i.e., the residence). Indeed, it is the extensive quality of space use that forces the differentiation of space by location, as space is scarce at locations and can accomodate only so much activity. Such differentiation is reflected in variable prices for space, or "land," viewed as a primary input to activity. As such an input, land is presumed to be a homogeneous good, like other goods in the traditional scheme. In this chapter, then, we will take up the movement over space in consumer behavior, leaving the extensive use of space by the consumer to the next section.

Journey-to-Work in the Labor-Leisure Model

In considering movement over space the logical place to begin is the journey-to-work, given the centrality of the work activity in time-inclusive models of consumer behavior. Perhaps the simplest case involves a given residence and place of work, with travel time and work time standing in some strict relationship. The archtypical case would have travel time and work time in a fixed proportion. This has realism only under the presumption of a fixed work shift time and a fixed trip duration, so that the proportionality of work and travel time is based on the necessity of a round trip for each shift worked. Obviously, the time variable for work and travel in such a case is really a proxy for frequency, but we will postpone explicit introduction of frequency for the moment. Activity times here will be gross rather than average.

Let us add travel cost k_d and time t_d of traversing the distance d in the journey-to-work to the money and time budgets, respectively, in the labor-leisure model:

$$Wt_w - x - k_d \geq 0$$

$$T - t_w - t_l - t_d \geq 0$$

Obviously, if these were constants independent of other times and expenditures, the loss of utility for external changes in their levels would be measured by the resource shadow prices:

$$\frac{dU}{dk_d} = \frac{\partial \mathcal{U}}{\partial x}\frac{dx}{dk_d} + \frac{\partial \mathcal{U}}{\partial t_w}\frac{dt_w}{dk_d} + \frac{\partial \mathcal{U}}{\partial t_l}\frac{dt_l}{dk_d} + \frac{\partial \mathcal{U}}{\partial k_d} = \frac{\partial \mathcal{U}}{\partial k_d} - \lambda$$

Likewise,

$$\frac{dU}{dt_d} = \frac{\partial U}{\partial t_d} - \theta$$

Note that provision is made here for inclusion of these arguments into the utility function, even when they are not choice variables for the consumer. The point is that treatment of consumer behavior should be exhaustive, covering all uses of time and money even if some are beyond the control of the consumer. Here the shadow prices have a negative sign, since increases in these arguments are depletions to stocks. Here the relative value of a marginal *reduction* in travel time versus a marginal *reduction* in travel cost is

$$\frac{- dU/dt_d}{- dU/dt_d} = \frac{\theta - \partial U/\partial t_d}{\lambda - \partial U/\partial k_d}$$

This dollar value of time *in travel* equals the general dollar of time θ/λ only with neutrality toward travel time and cost.

The cases that we are working up to will treat travel as an activity at least partially under the control of the consumer, with travel time and cost in the utility function as a general case. As stated above, the simplest situation would be that with proportionality between total travel time and total work time -

$$t_d = \alpha t_w$$

- with $\alpha > 0$ the constant of proportionality. For simplicity, assume, too, that travel cost is proportional to travel time:

$$k_d = \beta t_d$$

The budget equations for the labor-leisure become, on substitution,

$$(W - \beta\alpha)t_w - x \geq 0$$
$$T - (1 + \alpha)t_w - t_1 \geq 0$$

t_d and k_d can be eliminated from the utility function by substitution, so that the optimization problem is in form strictly analogous to the original labor-leisure problem. The differences are a revised utility function, so that the marginal utilities could have different functional forms, and different weights on t_w in the budget constraints.

Without writing out all of the first-order equations here, the dollar value of time can be given as

$$\frac{\theta}{\lambda} = \frac{\partial \mathcal{U}/\partial t_l}{\partial \mathcal{U}/\partial x} = \frac{W - \beta\alpha}{1 + \alpha} + \frac{\partial \mathcal{U}/\partial t_w}{\lambda(1 + \alpha)}$$

The marginal utility of work includes the effects of travel time and cost, effects that are usually thought to yield disutility.[1] Even with neutrality to work and travel, so that the partial vanishes, the dollar value of time have should be significantly less than is value without travel.

Changes in utility with marginal changes in the travel parameters α and β are

$$\frac{dU}{d\alpha} = -(\theta + \lambda\beta)t_w + \frac{\partial \mathcal{U}}{\partial \alpha}$$

$$\frac{dU}{d\beta} = - \alpha\lambda t_w + \frac{\partial \mathcal{U}}{\partial \beta}$$

where the partials with respect to α and β must be included because they entered the utility function by substitution. If they are negative, so are the marginal utilities of the changes. If they vanish (neutrality toward travel), then the relative value of a marginal reduction in the travel time compared to that of a marginal reduction in the unit money cost for travel is

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[1] On the other hand, there could be some positive utility from travel expenditures as a status item, although this seems far-fetched.

$$\frac{-\,dU/d\alpha}{-\,dU/d\beta} = \frac{1}{\alpha}\left(\frac{\theta}{\lambda} + \beta\right)$$

Presuming that $t_w > t_d$, so that $\alpha < 1$, this value is greater than the dollar value of time in general, and it may be taken as a measure of the dollar value of time in travel.

These simple examples assume that there is but a single route/mode combination available, or else that the same one will be chosen under any circumstance, and that the places of residence and work are fixed. Moses and Williamson presented an early model for discrete choices having much generality.[2] That generality also limits its usefulness in theorem derivation, however. Their approach is to specify additional time and money budget constraints for each mode/route combination available to the consumer for the journey-to-work. Thus, if there are n such alternatives, $j = 1, \ldots, n$, there would be n pairs of constraints in the labor-leisure model

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[2] Leon N. Moses and Harold Williamson, Jr., "Value of Time, Choice of Mode, and the Subsidy Issue in Urban Transportation," Journal of Political Economy, Vol. 71, June 1963.

$$Wt_w - x - k_j \geq 0$$
$$T - t_w - t_l - t_j \geq 0$$

where the k_j and t_j are constants giving the travel cost and time, respectively, for route/mode combination j. The shadow prices for the pairs of constraints are the set of pairs, λ_j and θ_j.

The first order conditions for leisure and work time and consumption expenditures are

$$\frac{\partial U}{\partial t_l} - \sum_j \theta_j \leq 0 \qquad \begin{cases} = 0 \text{ if } t_l \geq 0 \\ < 0 \text{ if } t_l = 0 \end{cases}$$

$$\frac{\partial U}{\partial t_w} - \sum_j \theta_j + W\sum_j \lambda_j \leq 0 \qquad \begin{cases} = 0 \text{ if } t_w \geq 0 \\ < 0 \text{ if } t_w = 0 \end{cases}$$

$$\frac{\partial U}{\partial x} - \sum_j \lambda_j \leq 0 \qquad \begin{cases} = 0 \text{ if } x \geq 0 \\ < 0 \text{ if } x = 0 \end{cases}$$

The first order conditions for a route/mode combination j are the budget constraints:

$$x + k_j - Wt_w \leq 0 \qquad \begin{cases} = 0 \text{ if } \lambda_j \geq 0 \\ < 0 \text{ if } \lambda_j = 0 \end{cases}$$

$$t_l + t_w + t_j - T \leq 0 \qquad \begin{cases} = 0 \text{ if } \theta_j \geq 0 \\ < 0 \text{ if } \theta_j = 0 \end{cases}$$

If a route is chosen, its shadow prices are positive; otherwise, they are zero. If the time and price pairs are all different, then only one route/mode combination should be chosen. The results thus obtained are basically those of the simple labor-leisure model with constant travel cost and time, there being but one pair of positive shadow prices λ and θ. This discrete choice formulation is not unlike the structure developed for travel selection in the River Basin Model. Moses and Williamson use this technique as a basis for discussing "diversion prices" for influencing changes in travel choices. However, unless solutions were worked out for each route/mode combination as if it were the only choice available, so that shadow prices and utility levels could be compared, there is very little we can say about the nature of the optimal solution here.

By way of comparison, suppose that route/mode choice were continuous --that is, suppose it were possible to trade travel time against travel cost in a continuous fashion according to some functional relationship between the two. For example, let

$$t_d - \mathfrak{T}(k_d) \geq 0$$

be added to the system as a constraint. Here the function $\mathfrak{T}$, presumed a negative monotonic function of k_d, gives the travel time required on the mode/route choice costing k_d. The idea is that a higher cost gains admittance to a faster mode, the fiction being that there are so many choices available as to present a continuum. The constraint is written as an inequality, since one may always waste time in travel and take more time than is necessary.

With the "continuous route/mode" choice in the simple leisure-labor model, there would be the additional first order conditions

$$\frac{\partial \mathcal{U}}{\partial t_d} - \theta + \phi \leq 0 \qquad \begin{cases} = 0 \text{ if } t_d \geq 0 \\ < 0 \text{ if } t_d = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial k_d} - \lambda - \phi \frac{\partial \mathcal{J}}{\partial k_d} \leq 0 \quad \begin{cases} = 0 \text{ if } k_d \geq 0 \\ < 0 \text{ if } k_d = 0 \end{cases}$$

Note that this problem, like the Moses-Williamson problem, treats the travel choice independently of the work time allocation. Here $\phi \geq 0$ is the shadow price on the travel constraint. It will be zero if at the optimum the consumer enjoys travel so much that he spends more time than is necessary on it (the marginal utility of travel must then be positive and equal to θ). If there is disutility to travel time, so that the marginal utility of travel time is negative, then ϕ must be positive. Finally, when there is complete neutrality to travel (so that partials with respect to time and cost vanish), optimal mode/route selection will be related to the dollar value of time through (in terms of k_d and t_d) as

$$\frac{\theta}{\lambda} = - \frac{\partial \mathcal{J}}{\partial k_d} > 0$$

the partial of $\mathcal{J}$ being negative. In such a fictitious situation as this, we would have precise valuation of marginal preferences and valuations concerning travel alternatives, something we do not have in the discrete choice case.

Trip Frequency in Route and Mode Selection

There is considerable loss of clarity in the models above with the failure to make frequencies explicit. For example, the Moses-Williamson journey-to-work route/mode choice problem can be restated to involve the number of trips, f_j, by route/mode type j as an explicit choice variable, with fixed parameters α_j and β_j as the time and money costs per trip, respectively. Then

$$t_j = \alpha_j f_j$$

$$k_j = \beta_j f_j$$

give the total time and money allocations to mode/route alternative j, used f_j times. Total travel time and cost over all alternatives are the sums of these components:

$$t_d = \sum_j t_j = \sum_j \alpha_j f_j$$

$$k_d = \sum_j k_j = \sum_j \beta_j f_j$$

These can be entered directly into the time and money budget constraints without having to specify separate budget constraints for each alternative. However, if there is some minimal number of trips f which must be taken, then there should be an additional constraint

$$\sum_j f_j - f \geq 0$$

Then, in the simple labor-leisure model, along with the usual first order conditional for t_l, t_w, and x will be the set of conditions for the frequencies f_j of using mode/route selection j:[3]

$$\frac{\partial \mathcal{U}}{\partial f_j} - \lambda \beta_j - \theta \alpha_j + \phi \leq 0 \qquad \begin{cases} = 0 \text{ if } f_j \geq 0 \\ < 0 \text{ if } f_j = 0 \end{cases}$$

where ϕ is the shadow price on the trip frequency constraint.

Obviously, some trips must be made in this model. With positive marginal utility to some trip type, there could be more trips than necessary, so that $\phi = 0$.

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[3] Variables t_j and k_j are presumed replaced in the utility function with f_j by substitution. The f_j may be in the utility function by their own right as well, but their marginal utilities must now also reflect time and cost effects.

More than likely, however, ϕ will be positive. It is apparent that more than one trip type may be utilized at the optimum, but it is perfectly possible for one to be so far preferable to others that it is the only one taken. In the case of neutrality to trips by whatever mode, then there will be only one positive trip frequency. With neutrality to travel, ϕ will be positive, so that that trip frequency would equal f. Then the ratio ϕ/λ gives the dollar value of trip frequency,

$$\frac{\phi}{\lambda} = \beta_j + \alpha_j \left(\frac{\theta}{\lambda} \right)$$

where j is the optimal route/mode. However, if $\phi > 0$, then a reduction in the frequency f (viewed as a "stock") is desired, since

$$\frac{dU}{df} = \frac{\partial \mathcal{U}}{\partial x} \frac{dx}{df} + \frac{\partial \mathcal{U}}{\partial t_l} \frac{dt_l}{df} + \frac{\partial \mathcal{U}}{\partial t_w} \frac{dt_w}{df} + \sum_j \frac{\partial \mathcal{U}}{\partial f_j} \frac{df_j}{df}$$

$$= - \phi < 0$$

This means that more trips are required than would be chosen otherwise.

For a mode/route selection j that is utilized, however many are utilized, the marginal utilities of increases in the time and cost parameters are given as

$$\frac{dU}{d\alpha_j} = -\theta f_j + \frac{\partial \mathcal{U}}{\partial \alpha_j}$$

$$\frac{dU}{d\beta_j} = -\lambda f_j + \frac{\partial \mathcal{U}}{\partial \beta_j}$$

The partials with respect to the arguments must be included, since they entered the utility function by substitution (see footnote 3 above). If these partials are negative, so are the marginal utilities of the increases. With neutrality toward the α_j and β_j, then the relative value of these changes is given by the dollar value of time

$$\frac{dU/d\alpha_j}{dU/d\beta_j} = \left(\frac{\theta}{\lambda}\right)$$

No clear statement of this type of relationship is possible in the Mosses-Williamson formulation.

The trip frequency version of the route/mode choice brings out the need to state explicitly the relationship between travel and work time, something we have not done adequately yet. Moses and Williamson's presentation treats travel time and cost as independent of work time, implying a work frequency equal to trip frequency but independent of work time. As suggested earlier, the logical relationship is one in which work frequency f_w represents the number of shifts worked. Therefore, in this model we can replace the constant f in the frequency constraint with the choice variable f_w. If we optimize with respect to f_w as well as t_w and the other variables in the labor-leisure model, plus the trip frequency by mode and route, we get the additional first order condition (presuming f_w is an argument to the utility function)

$$\frac{\partial U}{\partial f_w} - \phi \leq 0 \qquad \begin{cases} = 0 \text{ if } f_w \geq 0 \\ < 0 \text{ if } f_w = 0 \end{cases}$$

Obviously, to get positive f_w here requires positive marginal utility if $\phi > 0$, which means that the consumer prefers to break work time t_w up into separate episodes even though it requires more trips than he would otherwise make. If trip-making is so pleasurable that $\phi = 0$, the solution for f_w is at a satiation point relative to the other arguments.

An alternative specification is to assume that work shift length is fixed at s time units. Then $t_w = sf_w$ can be substituted into the utility function and the money and time equations. The revised first order condition for f_w is

$$\frac{\partial \mathcal{U}}{\partial f_w} - S(\theta - \lambda w) - \phi \leq 0 \qquad \begin{cases} = 0 \text{ if } f_w \geq 0 \\ < 0 \text{ if } f_w = 0 \end{cases}$$

remembering that the condition for t_w is now meaningless and that the marginal utility of f_w incorporates effects of the utility of work time. In this specification it is no longer necessary for the marginal utility of f_w (with time effects) to be non-negative to get a positive solution. With such pleasure to travel that $\phi = 0$, we have the traditional result expressed in frequency terms, such that neutrality to work produces the familiar $\theta/\lambda = W$.

In this version, the dollar value of time can be related to work frequency and trip frequency in mode/route choice j under the presumption that $\phi > 0$ according to

$$\frac{\phi}{\lambda} = \frac{1}{S + \alpha_j}\left\{(SW - \beta_j) + \frac{1}{\lambda}\left(\frac{\partial \mathcal{U}}{\partial f_w} + \frac{\partial \mathcal{U}}{\partial f_j}\right)\right\}$$

where, if j is the only alternative chosen, then $f_w = f_j$ since $\phi > 0$. If the partials vanish the dollar value of time equals the ratio of the net income after transportation earned per trip to the total time (work plus travel per trip. The marginal utility of an external increase in the required shift length S is

$$\frac{dU}{dS} = (\lambda W - \theta) f_w + \frac{\partial U}{\partial S}$$

where S entered the utility function through substitution for t_w.

Trip Frequency for Leisure Activities

In the models above with travel frequency explicit, it is apparent that when $\phi = 0$ and the number of trips exceeds that required for the journey-to-work, then there is the occurrence of trip-making for pleasure. The present treatment makes no distinction between such purposes, but it is simple enough to separate the journey-to-work from pleasure trips and enter both in the utility function. In such a case the frequency constraint on the journey-to-work should be a definitional equation, so that ϕ could take either sign. This is equivalent to stating simply that there cannot be a journey-to-work without an occurrence of work, which seems logical enough.

Frequencies for pleasure trips need not be constrained in such a manner. Johnson, for example, treats pleasure trip frequencies exactly as DeVany treats goods, with proportional time and cost coefficients.[4] That obviously makes them specific to origin and destination. Chiswick disaggregates them by type to allow specification by route and mode as well as purpose and origin and destination, although he does not do this explicitly.[5] Neither, however, faces up to the problem of distinguising from the properties of trips and the properties of the activities which take place at their destinations. If we thus allow consumers to spend time at destinations of trips before returning home, then the study of trip frequency behavior can be integrated with the disaggregation of out-of-home leisure activities by type. Finally, there could be differentiation among these out of home leisure activities according to pricing policies. Use of commercial facilities may be possible at a flat admission rate per occurrence (e.g., a swimming pool or a zoo), at an admission rate with a time limit (e.g., a movie), at a fixed rate per hour

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[4] M. Bruce Johnson, "Travel Time and The Price of Leisure," Western Economic Journal, Vol. 4, Spring 1966, pp. 135-45.

[5] Barry R. Chiswick, "The Economic Value of Time and The Wage Rate: A Comment," Western Economic Journal,

(e.g., a tennis court), or at some combination of these. Free activities would include visiting in houses and use of public facilities for which no charge is made.

By way of illustration, let leisure time t_l be disaggregated into activity times spent in leisure activities at home, t_m, $m = 1, \ldots,$ and times t_k spent in out-of-home activities, $k = 1, \ldots$ Let their corresponding frequencies of occurrence be f_m and f_k. It will be presumed that use of consumption goods x_i, $i = 1, \ldots,$ occurs entirely in home in the activities m. This is not necessary but makes things simpler. Let pleasure trips be undertaken for the purpose of their out-of-home leisure activities, on the presumption that travel is homogeneous with respect to trip purpose. This will be reconsidered below. f_d will denote the number of pleasure trips taken. The assumption will also be made that all activities occur at a single place, for example, the "city center," so that trip time α and cost θ per trip are constant. Likewise, a single route/mode alternative is assumed. Generalization to route/mode choice could be introduced in the manner above. Generalization to other activity locations will be brought up later. Finally, to make these results comparable to those in the discussion of activity frequency in the previous chapter, leisure out-of-home activity times here will be average times $\bar{t}_k$. The analysis to follow will not explicitly consider work time, the journey-to-work, or in-home activities. In general, their treatment would be much the same as in the specifications considered above and in the last section of the previous chapter.

In summary, then, utility will be defined here over activity times $\bar{t}_k$ and frequencies f_k (and f_d for travel; since $t_d = \alpha f_d = \bar{t}_d f_d$, $\bar{t}_d = \alpha$ and is a constant, although it does appear in the utility function through substitution). The following expressions will be introduced to the budget constraints:

$\sum_k (a_k + r_k \bar{t}_k) f_k$	direct money charges for leisure activities out-of-home
βf_d	travel costs for that leisure
$\sum_k \bar{t}_k f_k$	time spent on out-of-home leisure activities

αf_d	travel time for that leisure

where α is travel time per trip
β is travel cost per trip
q_k is admission price for k
r_k is unit time price for k

As before, trip frequency must have a constraint in addition to the time and money budgets:

$$f_d - \sum_k f_k \geq 0$$

The first order conditions which result for out-of-home leisure activities are

$$\frac{\partial \mathcal{U}}{\partial f_k} - \lambda(q_k + r_k\bar{t}_k) - \theta\bar{t}_k - \phi \leq 0 \begin{cases} = 0 \text{ if } f_k \geq 0 \\ < 0 \text{ if } f_k = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial \bar{t}_k} - \lambda r_k f_k - \theta f_k \leq 0 \qquad \begin{cases} = 0 \text{ if } \bar{t}_k \geq 0 \\ < 0 \text{ if } \bar{t}_k = 0 \end{cases}$$

and for travel frequency

$$\frac{\partial \mathcal{U}}{\partial f_d} - \lambda\beta - \theta\alpha + \phi \leq 0 \qquad \begin{cases} = 0 \text{ if } f_d \geq 0 \\ < 0 \text{ if } f_d = 0 \end{cases}$$

where ϕ is the shadow price on the frequency constraint. Its interpretation is analogous to that given above for the frequency constraint in mode/route choice for the journey-to-work.

Assuming that an activity k is undertaken, the (negative of) the marginal rate of substitution for frequency for average time in that activity is

$$\frac{\partial \mathcal{U}/\partial f_k}{\partial \mathcal{U}/\partial \bar{t}_k} = \frac{\bar{t}_k}{f_k} + \left(\frac{q_k + \phi/\lambda}{r_k + \theta/\lambda}\right)\frac{1}{f_k}$$

The term in parentheses on the right gives a ratio of full prices for frequency and time spent in the activity, being the sums of their "market" prices (if there are any) and the shadow prices on the resource used (with trips viewed as a stock allocated to activities). The dollar value of a trip as a resource is given by the first order condition for trip frequency as a function of trip cost (direct plus time valuation) less the marginal utility of trip frequency;

$$\frac{\phi}{\lambda} = g + \alpha\left(\frac{\theta}{\lambda}\right) - \frac{\partial \mathcal{U}}{\partial f_d}$$

For $\phi > 0$ (no extra pleasure trips), that marginal utility could well be negative.

Note that for free activities to occur their marginal utilities must be set equal to 0. With satiation in such activities, they could occur with $\phi = 0$. However, if any one has strictly positive marginal utility, then ϕ must be positive, so that no extra trips are made. That would mean that even if trips were pleasurable, the free activity would be performed enough to assure that no extra trips need be taken to satisfy the (relative) desire for travel.

An additional complication which might arise for activities charging admission (frequency) prices q_k but not time prices ($r_k = 0$) is that there may be for such activities a maximum limit on the duration of an occurrence. A movie would be an example. In terms of gross activity time the constraint would be

$$\gamma_k f_k - t_k \geq 0$$

or, on the average,

$$\gamma_k - \bar{t}_k \geq 0$$

There would be a shadow price $\psi_k \geq 0$ for every activity with such a constraint, and the marginal rate of substitution relation would become

$$\frac{\partial U/\partial f_k}{\partial U/\partial \bar{t}_k} \quad \frac{q_k + \gamma_k(\frac{\theta}{\lambda}) - (\frac{\phi}{\lambda})}{(\frac{\theta}{\lambda})f_k + (\frac{\psi_k}{\lambda})}$$

with ψ_k/λ as the dollar cost of the average time constraint. The multiplicity of these constraints makes analytic solution of the problem considerably more complicated.

A difficulty with the specification above is the assumption of homogeneity of trips regardless of purpose. This means, for example, that if there are extra pleasure trips made, so that $\phi = 0$, then all types of trips are equally pleasurable. The simple solution is to disaggregate trips by purpose (destination activity) and enter them separately in the utility function. This means replacing the single trip frequency constraint with a constraint for each activity

$$f_{dk} - f_k \geq 0$$

where f_{dk} is the number of trips made to perform f_k episodes of activity k. If this relation is forced to be a definitional equation, presuming that a trip cannot be made for an activity without an occurrence of the activity, then the f_{dk} arguments can be eliminated from the problem by substitution, remembering that activity frequency now incorporates trip preference in its utility relations. This is acceptable under fixed activity locations and mode/route selections. Here, then, travel time and cost parameters per trip, α_k and s_k, must be specific to activity. Then the first order conditions for out-of-home leisure activities (without average time constraints) are

$$\frac{\partial \mathcal{U}}{\partial f_k} - \lambda(q_k + s_k + r_k\bar{t}_k) - \theta(\bar{t}_k + \alpha_k) \leq 0 \qquad \begin{cases} = 0 \text{ if } f_k \geq 0 \\ < 0 \text{ if } f_k = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial \bar{t}_k} - \lambda r_k f_k - \theta f_k \leq 0 \qquad \begin{cases} = 0 \text{ if } \bar{t}_k \geq 0 \\ < 0 \text{ if } \bar{t}_k = 0 \end{cases}$$

There is no longer a general travel argument f_d. The new marginal rate of substitution relations are

$$\frac{\partial \mathcal{U}/\partial f_k}{\partial \mathcal{U}/\partial \bar{t}_k} = \frac{\bar{t}_k}{f_k} + \left(\frac{q_k + s_k + \alpha_k\,(\theta/\lambda)}{r_k + (\theta/\lambda)}\right)\frac{1}{f_k}$$

where the term in parentheses gives the ratio of the revised "full" prices.

This interpretation of the problem, incorporating the effects of travel into the utility of the activity, is a familiar one in the literature on travel behavior, although such trip-activity frequency and time relations are virtually never made explicit. As mentioned before, it would be possible to introduce route and mode choice for each activity in the manner presented for the journey-to-work, although the resulting optimization problem would become formidable. Likewise, the work activity and its journey can be explicit components. With average time constraints the shadow prices ψ_k will appear again.

Pleasure trips for their own sake have dropped out of the model but may be reintroduced quite simply and made specific to location. Thus, let f_{dk} now represent the number of trips to location k as round trip excursions purely for the ride. The utility of this trip need have nothing to do with the activity associated with location k, although its time and cost parameters are the same as those for the trip for the activity, being α_k and β_k, respectively. Then

$$\frac{\partial \mathcal{U}}{\partial f_{kd}} - \lambda\beta_k - \theta\gamma_k \leq 0 \qquad \begin{cases} = 0 \text{ if } f_{kd} \geq 0 \\ < 0 \text{ if } f_{kd} = 0 \end{cases}$$

Not only must the trip have positive marginal utility to occur, but that marginal utility must be greater than the marginal utility of the frequency f_k of the activity at k, since, by substitution, if f_{dk}, $f_k > 0$, then

$$\frac{\partial \mathcal{U}}{\partial f_{dk}} = \frac{\partial \mathcal{U}}{\partial f_k} + \frac{\partial \mathcal{U}}{\partial \bar{t}_k} \frac{\bar{t}_k}{f_k} + \lambda q_k$$

(assuming that f_k and $\bar{t}_k$ have positive marginal utility). Otherwise, it would be better either to do the activity at k on the trips or else not to make those trips.

Multi-Purpose Trip-Making

The presumption in these developments so far concerning trip-making is that all trips are single-purpose round trips and that only one occurrence of an activity takes place per trip. The complexity of the problem increases manyfold when these presumptions are relaxed. No attempt will be made here to do this for a "general case." It should be mentioned, however, that "composite activity trips" could be defined in a manner analogous to the definition of composite goods, on the assumption of fixed proportions of times in the component non-travel activities. Then the activity-trip combination could be treated as above.

Another approach is to define the multi-purpose trip as a distinct type of trip without forcing any particular relation among activities on the trip other than the requirement that one occurrence of each take place whenever the multi-purpose trip is made. As a simple example, let's look at a particular version of the simple labor-leisure model. Let there be the work activity and a leisure activity with frequencies f_w and f_l, respectively. Following an earlier specification, let the work shift length per occurrence be fixed at s and remunerated at rate w per unit time. Further, let the leisure activity be the generalized "costless" leisure of the classical model, with consumption expenditures x being a separate item. Finally, let there be three types of trips, k = 1, 2, 3, where

f_1 = frequency of trips for leisure only
f_2 = frequency of trips for work only
f_3 = frequency of trips for work and leisure

The locations for work and leisure are given but may be different. Therefore, let the time and cost parameters for the trip of type k be denoted α_k and β_k, respectively.

The labor-leisure optimization problem is now to

$$\text{Maximize} \qquad U = \mathcal{U}(f_k\ f_l,\ f_w,\ \bar{t}_l,\ x)$$

$$\text{Subject to} \qquad f_k,\ f_l,\ f_w,\ \bar{t}_l,\ x \geq 0$$

$$wsf_w - \sum_k \beta_k f_k - x \geq 0$$

$$T - \sum_k \alpha_k f_k - sf_w - \bar{t}_l f_l \geq 0$$

$$(f_1 + f_3) - f_l \geq 0$$

$$(f_2 + f_3) - f_w \geq 0$$

The last two constraints are necessary to assure that the activity occurrences are matched by the appropriate number of trips. Denote their shadow prices ϕ_l and ϕ_w, respectively. The first order constraints for a maximum are

$$\frac{\partial \mathcal{U}}{\partial x} - \lambda \leq 0 \qquad \begin{cases} = 0 \text{ if } x \geq 0 \\ < 0 \text{ if } x = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_w} + \lambda ws - \theta s - \phi_w \leq 0 \qquad \begin{cases} = 0 \text{ if } f_w \geq 0 \\ < 0 \text{ if } f_w = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_1} - \theta \bar{t}_1 - \phi_1 \leq 0 \qquad \begin{cases} = 0 \text{ if } f_1 > 0 \\ < 0 \text{ if } f_1 = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial \bar{t}_1} - \theta f_1 \leq 0 \qquad \begin{cases} = 0 \text{ if } t_1 > 0 \\ < 0 \text{ if } \bar{t}_1 = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_1} - \lambda \beta_1 - \theta \alpha_1 + \phi_1 \leq 0 \qquad \begin{cases} = 0 \text{ if } f_1 > 0 \\ < 0 \text{ if } f_1 = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_2} - \lambda \beta_2 - \theta \alpha_2 + \phi_w \leq 0 \qquad \begin{cases} = 0 \text{ if } f_2 > 0 \\ < 0 \text{ if } f_2 = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_3} - \lambda \beta_3 - \theta \alpha_3 + \phi_1 + \phi_w \leq 0 \qquad \begin{cases} = 0 \text{ if } f_3 > 0 \\ < 0 \text{ if } f_3 = 0 \end{cases}$$

On the assumption of no pleasure trips per se other than what is implicit in the generalized leisure activity, the two frequency constraints will be definitional equations. These could be used to eliminate f_1 and f_w from the problem, but since we will be interested in the interpretation of the utilities of the f_k, that will not be done here. With these constraints as equations, the shadow prices ϕ_1 and ϕ_w could take either sign, positive if the frequency of trips for an activity yields relatively more utility than does the frequency of the activity itself, negative otherwise.

No attempt will be made here to explore all possible cases. It should be apparent that different trip-making patterns are possible according to the nature of the utility function. In general, if all three types of trips are undertaken, then there is the relation

$$\frac{\theta}{\lambda} = \frac{(\beta_1 + \beta_2) - \beta_3}{\alpha_3 - (\alpha_1 + \alpha_2)} + \frac{1}{\lambda}\left(\frac{\partial \mathcal{U}/\partial f_3 - (\partial \mathcal{U}/\partial f_1 + \partial \mathcal{U}/\partial f_2)}{\alpha_3 - (\alpha_1 + \alpha_2)}\right) > 0$$

Since it is unlikely that the joint purpose type of trip would have less cost but greater time (or greater cost but less time) than the sums of the respective costs and times of the two single purpose trips, the right-most term would have to be sufficiently positive to make the whole expression positive (the first term on the right being negative). Thus the relationship between the marginal utilities of the frequencies here must be in the same direction (yield the same sign) as their time parameters and in the opposite direction of their cost parameters, if all three types of trips are to be undertaken. This would be reversed if the supposed relationship between times and costs for multi- versus single purpose trips does not hold.

With neutrality to all trips, so that the marginal utilities vanish everywhere, it is not likely that the equation above can hold, so that at least one type of trip is not made. It also means that the shadow prices on trip frequencies will be positive and can generally be solved for as functions of θ and λ. If there is also neutrality toward work, then $\phi_w = s(\lambda w - \theta)$ so that the dollar value of time θ/λ is less than the wage rate.

If there is not neutrality to trip-making, it is germaine to ask what the interpretation of the utility of the dual purpose might be vis-a-vis the utilities of single purpose trips. One view is that there may be reflected in the utility of the dual (multi-) purpose trip some effect due to the sequencing of those activities on that trip over and above the utility of the travel per se. In the example above, there could be deviation of the (marginal) utility of the dual-purpose trip from what might accrue simply to a weighted sum of the single purpose trips, such deviation being due to an altered salience of the activities when performed in such tight sequence, i.e., without the usual interruption associated with the single purpose trip set. Such a "mix effect" is only a proxy for sequencing effects, of course, and in this specification the effect is attributed to travel and not to dimensions of those activities themselves, as perhaps it should be. In any case, it is not thought unreasonable to attempt a better understanding of travel behavior by directly addressing in a model such as this the question of activity mix in (multi-purpose) trip pattern.

Some Applications in the Theory of Trip Frequency

Trip times, frequencies and costs are the stuff of transportation theory and modeling, of course, and there have been numerous empirical and theoretical investigations into the factors affecting trip behavior. Because of the complexity of that behavior, most of those investigations have turned from micro-study of the household to study of travel patterns at a higher level of aggregation. This has substantial advantage from a practical point of view, but often it does not reveal much about the social welfare implications of those travel patterns. Recently, however, there have been several renewed attempts to tie trip-making behavior back to micro-economic theory of consumer behavior, for small groups of households if not for individuals per se. There has always been interest in the dollar value of time as a normative index for use in assessing changes in the transportation system, and planners need to be able to trace the impact of planning decisions back through the whole activity structure of households in terms of time as well as money allocations and readjustments in facility use in the face of change.

There are three developments to be discussed here in this context. One concerns the application of the Lancaster activity-characteristics approach to the demand for "abstract" transportation modes. This is an attempt to construct a method for anticipating consumer reaction to the availability of a novel type of transportation mode, in terms of the mix of characteristics of that mode relative to the mixes of characteristics for existing modes. The second development is an attempt to find in a model of consumer trip-making behavior the origin of the "gravity law" for the distribution of trip ends in terms of the force of attraction of opportunities for activities over the friction of distance. The third development is an example of the derivation of similar entropy-type formulations for trip-making behavior among attracting opportunities in which marginal utilities of trips by purpose are made explicit and estimatable. These three developments all make or imply extensions of the micro-economic theory of behavior of individuals in frequency, time and money to the behavior of groups of such individuals. These illustrations, then, are of particular relevance to the River Basin Model, where just such an extension is made.

The study of the demand for travel by abstract modes is an attempt to present a theoretical framework in which demand for (public) travel modes can be analyzed in terms of the specific service characteristics of each. With travel demand by mode interpreted as demand for travel service components first, and thus for modes per se only indirectly, the demand for a non-existent or "abstract" mode can be estimated from statistical demand functions for existing modes in terms of their service characteristics by extrapolation. Moreover, by testing the sensitivity of demand to changes in the service mix of the abstract mode, we have an instance of the design of an "optimal" mode as a selection of service parameters in a continuous rather than a discrete choice of modes, much as discussed in a previous section of this chapter.

Although the original formulators of this approach, Baumol and Quandt,[6] did not present the individual consumer equilibrium position on which their method rests but focused instead on aggregate demand, it appears that the underlying model of individual consumer behavior is basically an application of a Lancaster-type utility model to the Moses-Williamson problem of route/mode selection in traveling between two given points.[7] For example, a simple model of this type could be a labor-leisure model stated in terms of frequencies in the manner of the development of this and the previous chapters, where uses of different transportation modes are viewed as separate activities generating characteristics in the Lancaster fashion as do other work and consumption activities. One version might be the choice of journey-to-work frequencies f_k by mode k, work frequency f_w for shifts of length s (so that gross work time is sf_w), leisure time t_l, and consumption expenditures x such as to

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[6] R. E. Quandt and W. J. Baumol, "The Demand for Abstract Transport Modes: Theory and Measurement," Journal of Regional Science, Vol. 6, Winter 1966, pp. 13-26

[7] Baumol and Quandt address themselves to inter-city trips, but the method is equally applicable to intra-city travel.

$$\text{Maximize} \qquad U = \mathcal{U}(z_m)$$

Subject to:

"production" of characteristics levels z_m in activities:

$$z_m = \sum_k e_{mk} f_k + e_{mw} f_w s + e_{ml} t_l + e_{mx} x$$

for each characteristic m = 1, . . .

budgets

$$T - \sum_k \alpha_k f_k - sf_w - t_l \geq 0$$

$$wsf_w - \sum_k \beta_k f_k - x \geq 0$$

frequency constraint for work trips

$$\sum_k f_k - f_w \geq 0$$

where $e_{m\cdot}$ are coefficients for the yielding of characteristics in activities

α_k, β_k are the time and cost of a work trip

w is the wage rate per unit time

s is the work shift length per occurrence, so that income $Y = swf_w$

For simplicity leisure here is a composite, at-home activity and the intermediate use of goods in activities after Becker and Lancaster has been omitted. Expansion of this model along other lines could follow the directions of previous sections, and no further analysis will be carried out here. Problems in obtaining analytical solutions to such problems were mentioned previously.

Baumol and Quandt presume that on the basis of such individual optimization behavior individual demand functions for the f_k can be derived and aggregated into class demand functions for those mode/route alternatives in terms of the various trip characteristics (service levels) coefficients e_{mk}, prices β_k, and times α_k.

Strictly speaking, other parameters of the optimization problem should also be arguments to the demand functions, but they are not specified by Baumol and Quandt, who, after all, do not specify the individual optimization problem. Here, such parameters would be w, s, and the other characteristics coefficients. In any case, the result would be the aggregate demand functions

$$f_k = \mathcal{D}_k(e_{m\hat{k}}, \alpha_{\hat{k}}, \beta_{\hat{k}}, Y)$$

where the arguments cover all subscripts $m = 1, \ldots$ and $\hat{k} = 1, \ldots, k, \ldots$.

To make the abstract mode application, Baumol and Quandt postulate that the functions $\mathcal{D}_k$ are the same for all modes. The reasoning is that if utility is defined directly over characteristics, then the only source of differences in demands for modes lies in differences in characteristics coefficients and time and cost parameters, the functional relations between these being the same for all travel demand functions. Thus, the general demand function $\mathcal{D}$ which results yields a demand estimate for any mode, real or abstract, on the insertion of the appropriate parameter values representing that mode. The function $\mathcal{D}$, then, treats the travel choice problem as a problem in selection among "continuous modes." The authors present some statistical results based on presumed forms of the demand function, adding other parameters to reflect characteristics of the origin and destination locations. Certain further modifications have been suggested in subsequent articles.[8]

The central weakness in this and many travel demand models is the independence of travel behavior from other activity behavior. This results precisely because the model-builders do not start with a comprehensive model of individual behavior. By using an expanded version of the model above such problems could be avoided. It seems intuitively obvious that changes causing shifts in travel demands will also cause changes in the activities for which those trips are made. Yet models which do not allow identification of the latter types of changes are not likely to represent adequately the former types. There are practical reasons for these omissions, of course. Unlike much of economic theory, trip theory is readily applied in problems of urban and regional transportation planning. Complete time budget information necessary for analyzing travel behavior in the context of non-travel behavior is extremely expensive to collect and analyze. Moreover, there are many problems yet to be resolved in activity definition and measurement. On the other hand, much current data on trip patterns by destination land use, purpose, and time of day could be recast in time budget form for out-of-home activities.

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[8] See Reuben Gronau and Roger E. Alcaly, "The Demand for Abstract Transport Modes: Some Misgivings," pp. 153-157, and R. E. Quandt and W. J. Baumol, "The Demand for Abstract Transport Modes: Some Hopes," pp. 159-162, in Journal of Regional Science, Vol. 9, April 1969.

As has been mentioned at several points, mode choice decisions of the type above could be extended to cover trips for different purposes within the metropolitan area, where the non-travel activities at destinations which define trip purpose have their own time and money components. The series of articles initiated by Niedercorn and Beckdolt on a utility model derivation of the "gravity law" of spatial interaction makes a start in this direction but still stops short of an integrated view of travel and non-travel behavior.

In their original article[9] Niedercorn and Bechdolt see utility as a function of both the numbers of trips from an origin to destinations (for unspecified purposes) and the number of "opportunities" available there, the latter being used to weight the former in the utility function. This has the consequence that the optimum number of trips to a destination tends to vary directly with the number of opportunities available there, as in the gravity law formulation. Unfortunately, Niedercorn and Bechdolt assume independent determination of the total time and total money to be budgeted to travel. This has the consequence that usually only one constraint - either the time budget or the money budget but not both - is binding at the optimum, whereas models examined in this presentation presume that any slack resource would tend to be put to use in some other (non-travel) activity. In any case, the authors show that with certain utility functions, the inverse power relation between trip frequency and distance (in terms of time or cost, depending on which constraint is binding) typical of the gravity law can result in utility maximization.

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[9] J. H. Niedercorn and B. V. Bechdolt, Jr., "An Economic Derivation of the 'Gravity Law' of Spatial Interaction," Journal of Regional Science, Vol. 9, August 1969, pp. 273-282.

In subsequent articles Mathur[10] and then Allen[11] suggested that the characteristics of the trips be introduced to the utility function in the manner of Lancaster's work. Indeed, this results in a formulation quite similar to that presented in connection with the Baumol and Quandt approach but without the work and leisure components. Unfortunately, the slack budget situation is perpetuated, because they presume, like Niedercorn and Bechdolt, a prior independent determination of the maximal amounts of time and money to go into travel. That a solution is possible with binding time and money constraints is demonstrated by the example presented in the appendix to this paper. The characteristics approach of Mathur and Allen allows greater detailing of travel phenomena, but it does not rectify the limitations of the specification of travel behavior as separable from non-travel behavior.

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[10] Vijay Mathur, "An Economic Derivation of the 'Gravity Law' of Spatial Interaction: A Comment," Journal of Regional Science, Vol. 10, December, 1970, pp. 403-405. See also Niedercorn and Bechdolt's "Reply" which follows, pp. 407-410.

[11] W. Bruce Allen, "An Economic Derivation of the 'Gravity Law' of Spatial Interaction: A Comment on the Reply," Journal of Regional Science, Vol. 12, April 1972, pp. 119-126. Again, Niedercorn and Bechdolt follow with "A Further Reply and A Reformulation," pp. 127-136.

The third development in this set of applications comes in some recent work of Beckmann, Golob and Gustafson.[12] It, too, focusses on trip frequencies and trip purpose in the context of time and money constraints and uses a specific (separable and additive) form of the utility function to lead to empirical statements about trip behavior. It does not incorporate the Lancaster approach to activity characteristics and is mentioned here primarily because of its method of handling binding time and money constraints. In particular, it represents a less than satisfactory (in this writer's view) way of determining shadow prices in the two constraint case.

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[12] Martin J. Beckmann, Thomas F. Golab, and Richard L. Gustafson, "An Economic Utility Theory Approach to Spatial Interaction," paper presented at the 1972 North American Meeting of the Regional Science Association, Philadelphia, November 11, 1972.

Beckmann *et. al.* set up the trip-making problem similar to the travel components of models presented in this paper and to the original Niedercorn-Bechdolt model in general form:

$$\text{Maximize} \qquad U = \mathcal{U}(f_{jk})$$

$$\text{Subject to} \qquad f_{jk} \geq 0$$

$$M - \sum_j \sum_k \beta_j f_{jk} \geq 0$$

$$T_d - \sum_j \sum_k \alpha_j f_{jk} \geq 0$$

where M and T_d are the money and time stocks available for travel, f_{jk} give the number of trips chosen to location j for purpose k (presumably a non-travel activity), and α_j and β_j are the travel time and cost to location j, respectively. This is a straightforward application of the basic consumer model to trip frequency but ignoring time and money aspects of non-travel behavior. The first order conditions for a maximum are

$$\frac{\partial \mathcal{U}}{\partial f_{jk}} - \lambda \beta_j - \theta \alpha_j \leq 0 \quad \begin{cases} = 0 \text{ if } f_{jk} \geq 0 \\ < 0 \text{ if } f_{jk} = 0 \end{cases}$$

The marginal rate of substitution between trips to the same location for different purposes is thus (minus) unity, while all those between trips to two given locations for whatever purposes are equal.

At this point, the authors make further simplifications and postulates. First, they imply that both constraints will be binding. Although this can of course happen, it is not likely with fixed stocks M and T_d unless those stocks stand in (near-) optimal proportions. Thus, the Niedercorn-Bechdolt assumption of a slack constraint may be more correct technically, given the specification of these models which omit non-travel behavior.[13] Second, they "merge" (my term) the cost parameters and shadow prices by defining the "total" interaction cost" r_k of a trip to a location as the weighted sum of α_k and β_k:

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[13] Actually, it is the time dimension of non-travel activity that is excluded. The utility of a frequency can reflect both the activity (purpose) and travel effects. When different effects are subsumed by the same arguments in a utility function, however, it is no simple matter to decide what type of function can adequately reflect all those effects.

$$r_k \equiv \left(\frac{\lambda}{\lambda + \theta}\right) \beta_k + \left(\frac{\theta}{\lambda + \theta}\right) \alpha_k$$

Then the first order conditions

$$\frac{\partial \mathcal{U}}{\partial f_{jk}} = \psi r_k$$

in the single shadow price $\psi = \lambda + \theta$ can be fitted statistically by traditional methods once a utility function is specified, presuming the existence of data for r_k.

The authors claim that r_k is specific only to origin and destination of the trip, which is true only if all people at the origin have identical preferences and stocks set aside for travel, for θ/λ will be specific to relative preferences and stocks. Yet they do not impose that restriction on preferences in their investigations. In their empirical work, where regressions are carried out by income class, origin, destination, and purpose (working and shopping), they actually use travel time for r_k, which implies that $\lambda = 0$. Moreover, they then go on to interpret ψ as the "marginal utility of income," which could happen only if $\theta = 0$ and $r_k = \beta_k$. Indeed, even under those conditions ($\psi = \lambda$), it would be the marginal utility of M, not income. M itself is never identified in the regressions, either as a parameter or control variable, nor is T_d. Thus it would seem that there is no way to check whether their empirical determination of parameter values would be consistent with the budget constraints.

Given this beginning, the method used by the authors to derive regression equations and obtain estimates of the values of marginal utilities is quite ingenious. However, because the simplifications of the theory as described above are considered inadequate that method will not be investigated further here.

All three developments presented here suffer badly in theory (and presumably also in application) from the isolation of travel from non-travel behavior and consequently also from the inadequate treatment of the budget constraints and their shadow prices. In passing, it is interesting to note that while the literature generally ascribes disutility to travel, these examples impute positive utility to trip frequency. This is not unreasonable if it is "purpose," not travel as such, which is desired. The characteristics approach helps to separate these effects, but it would seem that the inclusion of choice variables for the destination activities per se is by far the best way to clarify the situation.

To give an example of the way in which activity and travel behavior can be treated, one author (who asks not to be quoted) considers the problem of making trips to shop. Disaggregating consumption expenditures x into components x_k specific to (purchased at) locations k, he defines utility over the x_k, the trip frequencies to shop, f_k, and work and leisure time, t_w and t_l, respectively. The utility of x_k can include the effects of the goods per se and the time spent in shopping for them, since shopping time is assumed proportional to the goods bought. Likewise, the utility of the f_k can include travel effects and shopping frequency effects. Then the consumer's optimization problem is to

$$\text{Maximize} \qquad U = \mathcal{U}(x_k, f_k, t_w, t_l)$$

Subject to

$$T - t_w - t_l - \sum_k \alpha_k f_k - \sum_k \gamma_k x_k \geq 0 \quad \text{(time)}$$

$$wt_w - \sum_k \beta_k f_k - \sum_k p_k x_k - \sum_k r_k \left(\frac{x_k}{f_k}\right) \geq 0 \qquad \text{(money)}$$

where	β_k	is the cost of a trip to k
	α_k	is the time for a trip to k
	k	is the time to shop for a unit of the good x_k at k
	w	is the wage rate
	p_k	is the price of a unit of good k
	r_k	is the cost of storing a unit of good k

The novelty of this model is the assessment of costs for storing the amounts x_k of the goods purchased. Since f_k trips are made to purchase x_k, the average amount purchased each trip is, of course, x_k/f_k. If it is presumed that the same amount (this average) is purchased each trip, then no more than the average need ever be stored at home at one time. Hence, the average amounts are the arguments to storage costs in the money budget. Note that the storage cost rates r_k could be interpreted as costs of appropriate housing components. Generalization of the model to relate other activities to housing inputs would thus create a model for the demand for residential space, in a Becker-type format.

Such considerations will be brought up again in the next chapter, where this model will be reviewed in more detail. Although limited to treatment of only one type of destination activity – note that there are no trips for work or leisure – this example does show how time/money tradeoffs may be made between trip frequency and destination activity choice variables. For example, the (negative of) the marginal rate of substitution of frequency for a good is equal to the ratio of the marginal costs

$$\frac{\partial \mathcal{U}/\partial f_k}{\partial \mathcal{U}/\partial x_k} = \frac{(\theta_k - r_k x_k/f_k^2) + \alpha_k(\theta/\lambda)}{(p_k + r_k/f_k) + \gamma_k(\theta/\lambda)}$$

with time valued at θ/λ. Note that an increase in frequency means a savings in storage costs, thus the negative sign of the change in storage costs with an increase in frequency.

To incorporate all of the dimensions and activity choice specificities introduced in the various models of this and the previous chapter into a single model would result in an optimization problem of considerable proportions, and no attempt to do so will be made here. In many cases the first order conditions for arguments would be little changed, so that the separate models become but components of the larger one. In some cases, however, there may be substantial modifications required. The separate models serve our didactic purposes adequately, however, and we will not attempt to include them all in the discussion of the final topic to be addressed in the theory of individual consumer behavior, residential location and the use of space, to which we turn in the next chapter.

SECTION V
RESIDENTIAL LOCATION AND THE USE OF SPACE

While time and money allocations to activities are explicit choice variables to the social sector (households) in the River Basin Model, residential location and the purchase of residential space (housing) are not. Therefore, the discussion of these topics will be more cursory than treatment of other topics in the previous chapters, since optimizing behavior in residential location and use of space will be of less relevance to the critique to be undertaken. Moreover, it will be suggested that most of the models already discussed can be turned into location models by treating travel costs and time and housing (as one of the generalized goods inputs to leisure and activities) price as functions of distance or location, then maximizing utility over location. Choice of residential location and the space (housing) to be purchased there thus becomes a choice of the optimum optimorum — picking the best activity/consumption pattern from the optimal patterns for each location as derived previously for the general case (location).

For these reasons, the discussion to follow will not be as comprehensive as to activity pattern as previous discussions, although it should be emphasized that the optimal activity pattern is very much a function of residential location in these models. Indeed, the purpose of including this chapter is to suggest the importance of residential location in time and money allocations and to show how some theorists have approached residential location problems. Their mechanisms might find application in a revised simulation procedure for the Social Sector of the River Basin Model.

The Industrial Location Analogy

The general statement by Isard in 1956 of the problem of the choice of residential location as a function of shopping pattern marks the beginning of modern theory of residential location.[1] As presented by Isard, the consumer location problem is strictly analogous to the Weberian location problem for industrial plant location, where the objective involves finding a location central to sources of inputs for production such that the obtaining of inputs and the production and distribution of the product maximizes profit. The household, accordingly, is faced with finding a location central to sources of goods inputs (i.e., retail establishments) to its "production" of utility, which is to be maximized subject to budget constraints. In addition to direct outlays for goods, the consumer must bear the costs of procuring those goods as functions of distances traveled, frequencies of trips, and amounts of goods purchased.

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[1] Walter Isard, Location and Space Economy, (Cambridge: M.I.T. Press), 1956.

So far this model is similar to the shopping model of the last chapter, except that it lacks a time budget and does not introduce storage costs. What makes it different, of course, is the simultaneous choice of the residential location along with the pattern of goods purchased and trips made for the purchases. Rather than presenting Isard's original formulation, detailing the additional assumptions he makes for trip cost and frequency relations,[2] we will instead take up the shopping model of the last chapter which includes a time budget and storage costs and cast it in the form of a location model like Isard's. The results will not be too different and will contain the desired time elements. The first order conditions for a maximum for this model are

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[2] Isard makes trip frequency dependent on purchase level and travel costs incorporate implicitly some fixed value of time component.

$$\frac{\partial \mathcal{U}}{\partial t_w} + \lambda w - \theta \leq 0 \qquad \begin{cases} = 0 \text{ if } t_w \geq 0 \\ < 0 \text{ if } t_w = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_1} - \theta \leq 0 \qquad \begin{cases} = 0 \text{ if } t_1 \geq 0 \\ < 0 \text{ if } t_1 = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial f_k} - \lambda\left(s_k - r_k \frac{x_k}{f_k^2}\right) - \theta\alpha_k \leq 0 \qquad \begin{cases} = 0 \text{ if } f_k \geq 0 \\ < 0 \text{ if } f_k = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial x_k} - \lambda\left(p_k + \frac{r_k}{f_k}\right) - \theta \;\; k \leq 0 \qquad \begin{cases} = 0 \text{ if } x_k \geq 0 \\ < 0 \text{ if } x_k = 0 \end{cases}$$

The first two are the traditional relations of the labor-leisure model. It was pointed out previously that increased frequency can reduce storage costs but add to travel costs. Goods now have procurement and storage costs (the latter in terms of the average per trip) in addition to direct purchase price. The marginal utility of an external change (increase) in the storage requirement cost r_k is

$$\frac{dU}{dr_k} = \frac{\partial \mathcal{U}}{\partial t_w}\frac{dt_w}{dr_k} + \frac{\partial \mathcal{U}}{\partial t_l}\frac{dt_l}{dr_k} + \sum_{k'}\left(\frac{\partial \mathcal{U}}{\partial f_{k'}}\frac{df_k}{dr_k} + \frac{\partial \mathcal{U}}{\partial x_{k'}}\frac{dx_{k'}}{dr_k}\right)$$

$$= -\lambda\left(\frac{x_k}{f_k}\right)$$

To make this a location model after Isard, the travel time and cost parameters α_k and β_k must be considered as functions of the distances from the locations k to the residential location. If those locations can be identified in terms of a set of location variables or coordinates, then the distances can be written as functions of the location variables identifying the residential location. If l_j is one of those location variables, j = 1, . . . , then the optimal location can be derived by maximizing utility with respect to the l_j as well as other choice variables. Since the l_j are not included here in the utility function, the first order condition for l_j would be

$$\sum_k f_k\left(\left(\frac{\theta}{\lambda}\right)\frac{\partial \alpha_k}{\partial l_j} + \frac{\partial \beta_k}{\partial l_j}\right) = 0$$

$$j = 1, \ldots$$

Finding an analytic solution for the optimal location in terms of the l_j is no simple matter, and various approximation methods have been developed for Weberian-type problems,[3] though not in a time budget context.

Most models of residential location start with one dimensional space, or location along a line, and then generalize (usually informally) to two-dimensional space, where there may not be an analytic solution of a simple nature. For example, if in the shopping model the source locations k for goods are strung out along a line, with d_k being the distance from an end of the line to location k and d being the (variable) distance to the residential location, and if the travel times and costs are proportional to distances traveled —

$$\alpha_k = \alpha \left| d_k - d \right|$$

$$\beta_k = \beta \left| d_k - d \right|$$

— then there is a single first order condition for the optimal value of d:

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[3] See, for example, Leon Cooper, "An Extension of the Generalized Weber Problem," Journal of Regional Science, Vol. 8, Winter 1968, pp. 181-197.

$$-\sum_{k|d_k>d} f_k \left\{\left(\frac{\theta}{\lambda}\right)\alpha + \beta\right\}$$

$$+\sum_{k|d_k<d} f_k \left\{\left(\frac{\theta}{\lambda}\right)\alpha + \beta\right\} = 0$$

which is just

$$\sum_{k|d_k>d} f_k = \sum_{k|d_k<d} f_k$$

that is, the number of trips made in each direction is the same. It can be expected that the optimal location will be intermediate to the shopping locations. Even with the simple relation above, actual determination of optimal d may not be straightforward.

The possibility of extending a model such as this to include other out-of-home activities besides shopping suggests itself immediately, and indeed this will be considered in subsequent section of this chapter. First, however, let us turn to the problem of the actual use of space (land) at the residence along with selection of that residential location. This will be considered in the context of the simple labor-leisure model without the detailing of consumer activities and frequency relations.

Residential Space Use and Bid Price Curves

In the early work on residential location and space use following Isard's formulation, trip frequencies were not made explicit and goods were considered in an aggregated form, save for housing. Out-of-home activity was not specified by type; instead, it was simply presumed that expenditures for travel to the "center" along a line for whatever purpose must be made. The only variable in determining that travel cost was the distance to the residential location to the center. On the other hand, while trip pattern was aggregated, actual purchase of space, or "housing" or "land", for residential use was introduced. Variation in its price per unit over distance from the center thus replaced travel frequency as the central factor in selection of optimal residential location. In the Isard-type model, the collapsing of all destination opportunity locations to a single "center" would lead to optimal residential location at that center, there being no interplay of land prices and travel costs over distance.

It was Alonso who first formulated the tradeoff of residential space and distance through land prices and travel costs in the choice of the optimal residential location.[4] In his model utility was defined over housing (land) x_h and all other goods, the composite x_m. Transportation costs, k(d), were an increasing function of the distance d from the center, without specification of trip frequency or purpose. Housing (land) price per unit was likewise a function of distance, p(d), here a decreasing function.[5] Income Y is fixed.

Alonso's consumer optimization problem can be stated as

$$\text{Maximize} \qquad U = \mathcal{U}(x_m, x_h)$$

Subject to

$$Y - x_m - p(d)x_n - k(d) \geq 0$$

First order conditions for a maximum are

$$\frac{\partial \mathcal{U}}{\partial x_m} = \lambda$$

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[4] William Alonso, Location and Land Use, (Cambridge: Harvard University Press), 1964.

[5] Alonso subsequently demonstrates how a bidding procedure for land would lead to such a price schedule. See below.

$$\frac{\partial U}{\partial x_h} = \lambda p(d)$$

$$\frac{\partial U}{\partial d} \equiv 0 = \lambda \left\{ x_h \left(\frac{\partial p}{\partial d} \right) + \frac{\partial k}{\partial d} \right\}^{6}$$

where λ is the shadow price for money. At the optimal distance, the first order condition for distance — called the "location equation" — requires that the marginal savings for reduced land price with distance—

$$- x_h \frac{\partial p}{\partial d} > 0$$

— be set equal to the marginal costs of transportation at that location —

$$\frac{\partial k}{\partial d} > 0$$

(assuming no (dis-) utility to distance), where x_h is here the optimal amount of housing.

A provocative construct set up by Alonso is his "bid rent curve" for land. This curve is the function $\mathcal{P}(d)$ which gives the price the consumer would bid at any location d to be indifferent between location at d

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[6] Alonso notes that the marginal utility of d need not be definitionally zero if d is considered a proxy for travel in U.

and location at the center at some price p_o. That is, $\mathcal{P}(d)$ traces an indifference curve over land price with distance, relative to the price p_o at the center, which may be considered a parameter to $\mathcal{P}(d)$. $\mathcal{P}(d)$ takes into account variations in the optimal levels of x_h and x_m with distance. In fact, $\mathcal{P}(d)$ is precisely that function for which the first order condition

$$\frac{\partial \mathcal{P}}{\partial d} = -\frac{\partial k}{\partial d}\left(\frac{1}{x_h}\right) < 0$$

holds at all distances, x_h being the optimal for that distance, relative to a price p_o at the center and specific to a certain level of utility. $(1/\lambda)(\partial U/\partial d)$ may be added to the right-hand side if it is not definitionally zero.

Since a bid price curve is specific to a utility level and a price p_o at the center, the family of curves for various p_o defines a set of indifference curves for various utility levels. Utility will vary inversely with p_o, since a greater p_o means that at the center (when $d = 0$)

$$\frac{dU_{d=0}}{dp_0} = \frac{\partial \mathcal{U}}{\partial x_{mo}} \frac{dx_{mo}}{dp_0} + \frac{\partial \mathcal{U}}{\partial x_{ho}} \frac{dx_{ho}}{dp_0} = - \lambda x_{ho} < 0$$

where x_{mo} and x_{ho} are optimal levels when d is forced to be zero. hus if there is some market price structure $\mathcal{P}_m(d)$ for land, the consumer would maximize his utility by locating at the distance for which the price structure intersects the lowest possible of his bid price curves. In general, this would be a point of tangency from below of that bid price curve with the price structure, given that the necessary higher order conditions for such a tangency hold. By demonstrating that bid price curves may be steeper (more negative) for lower income groups than higher ones at most distances, Alonso explains spatial stratification of residence by income in rings around the center, with income usually incresing with distance. Using the bid price construct, Alonso develops a hypothetical bargaining procedure by which the market price structure itself might be generated.

It should be apparent that a time budget could easily be added to Alonso's framework in much the same way as it was introduced to non-residential models in the previous section. The simplest modification would be the introduction of leisure time t_1 to the utility function, the specification of a travel time function t(d), with $\partial t(d)/\partial d > 0$, and the addition of the time budget constraint

$$T - t_1 - t(d) \geq 0$$

The first order condition for leisure time is simply

$$\frac{\partial \mathcal{U}}{\partial t_1} = \theta$$

and the revised location equation can be written (in bid price form) as[7]

$$\frac{\partial \Phi}{\partial d} = - \frac{1}{x_h} \left\{ \frac{\partial k}{\partial d} + \left(\frac{\theta}{\lambda}\right) \frac{\partial t}{\partial d} \right\}$$

The inclusion of the marginal cost of travel time as the valuation of the marginal travel time at the dollar value of time (θ/λ) should make bid price curves steeper (more negative), all else equal. Note that (θ/λ) may vary with distance along a bid price curve. This is a particularly interesting phenomenon, for one wonders whether the dollar value of time should increase or decrease with distance. The answer obviously depends on the relative scarcity of time and money at a distance, and that is a matter of stock levels, prices and preferences there.

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[7] As before, (dis)utility to distance through travel may be added to this expression if appropriate.

To place residential location squarely in the context of the labor-leisure model, income could be allowed to vary through the choice of work time, t_w. Here work time would be explicit in the budgets for time and money and possibly also in the utility function. Then a change in location would entail not only changes in leisure and travel time and in money outlays for housing, other goods and travel, but it would also involve a change in the amount of time actually worked, thus the income stock. There would be a complementarity between optimal residential location and associated bid price curves and the labor offer schedule, for the optimal location as well as the labor offer will vary as the wage rate varies. In the traditional case of no (dis-) utility to work, the dollar value of time equals that wage rate and does not vary along a bid price.

Location by Occupation (Wage Rate)

It is apparent that the entire development of the previous chapters, in which the labor-leisure model was expanded to incorporate more detail in activity choice through time and money allocations, could be retraced and location equations derived for each model.[8] This would be tedious, and the general method for carrying out such derivations should become apparent here. We will focus instead on some developments in the literature which are relevant to the discussion of the River Basin Model, with some digression later on activity patterns. Despite the plea of previous chapters for the inclusion of frequency variables in time and money allocation models, no attempt will be made here to introduce frequency choice into each model considered. Again, the method of previous chapters should suggest how that might be done.

Wingo was perhaps the first to consider in theory the systematic effect of transportation time (through its implication for leisure time) as well as transportation cost on residential location.[9] What is of particular relevance for the River Basin Model is that his approach is a group location model, rather than one for the individual consumer, and it focuses on the role of occupation and wage structure in residential location. As such it gives a range of locations for the group, rather than the optimum location for any individual, and this turns out to create some inconsistencies in the model.

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[8] For example, the labor-leisure version of the Alonso model above could next be extended to cover the Moses problem for overtime work. Such a treatment has, in fact, just recently appeared in the literature. See Hiryuki Yamada, "On The Theory of Residential Location: Accessibility, Space, Leisure, and Environmental Quality," Papers of the Regional Science Association, Vol. 29, 1972, pp. 125-135.

In effect, Wingo presumes an Alonso-type location problem for the individual, with a time budget but holding actual work time and the time budget fixed. Instead of optimizing utility for the individual, he falls back on the labor offer curve as an independent function for computing a rent schedule over distance as a function of gross work time (work plus travel for the journey-to-work) and net wage (computed over gross work time), along with travel costs. This rent schedule represents the rents that could be charged workers (consumers) distributed around a work place. As such it deals directly with determination of the market price (rent) structure without locating individuals explicitly. However, to do this it must presume a type of partial optimization for the individual in terms of his work/leisure/travel relations, and the nature of this "optimization" is the weak point of his development.

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[9] Lowdon Wingo, Jr., Transportation and Urban Land, (Washington: Resources for the Future, Inc.), 1961, and "An Economic Model of the Utilization of Urban Land for Residential Purposes," Papers of the Regional Science Association, Vol. 7, 1961, pp. 191-205.

Wingo poses the location problem for a group of workers who are homogeneous, having the same preferences for consumption and leisure since they share a common labor offer curve. Their common employer wishes to hire them at a given shift length of $t_w = s$ hours per worker. Frequency of work and journey-to-work are implicit, but the implication is that they are equal and fixed. The questions are, what wage must the plant pay to get shifts of s from each worker — it must pay all workers the same wage — and what rent differentials will develop as these workers distribute themselves in space around the plant?

The classical wage theory has shown how a labor offer curve can be developed from a utility analysis in which income or consumption and leisure time are evaluated in terms of the contribution to the consumer's utility. Indeed, the labor offer curve, giving the equilibrium work time as a function of the wage rate, is a solution of the first-order conditions (including the budget constraints) for maximal utility. Wingo is interested in reversing the roles of these two components, wage rate and work time, by holding work time fixed and finding the equilibrium wage rate. Therefore, he takes the inverse function of the labor offer curve to yield

$$w = g(s)$$

Since preferences are assumed identical for all workers in the group, they all share this same view of the work time-wage relationship.

The first question Wingo raises concerns the wage the worker must be paid in order to give up leisure time for income when he must make a journey-to-work as well as work the shift of length s. At this point, Wingo makes an important assumption - that the function gives the remuneration necessary to induce the worker to yield a marginal unit of leisure. As we know, w equals the value of that remuneration only when there is no (dis-) utility to the argument of $\mathfrak{g}$. Moreover, w is the remuneration per unit time which must be applied to the argument of $\mathfrak{g}$ as representing all non-leisure time; otherwise it would be a function of more than one independent variable. This, in turn, implies indifference among uses of non-leisure time (which, in this case, all yield zero utility). Under these conditions $\mathfrak{g}$ becomes, as Wingo calls it, the "marginal value of leisure function," with non-leisure time as the argument.[10]

Consider, then, a worker living d units from his place of employment. With a journey-to-work taking t(d) time, he must yield s + t(d) units of leisure to complete s units of work. According to the marginal value of leisure function, he would be willing to do this if he received a "manifest wage" w(d) per unit time over the total non-leisure period, where

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[10] Note that although $\mathfrak{g}$ gives the marginal value of leisure only with no (dis-) utility to non-leisure, w as some function $\hat{\mathfrak{g}}$ can still be determined with such (dis-) utility to non-leisure; it just cannot be given that interpretation. Likewise, $\mathfrak{g}$ (or $\hat{\mathfrak{g}}$) need not be a function solely of non-leisure time as a whole but may have as arguments various components (such as work and travel) separately. In these cases, of course, $\hat{\mathfrak{g}}$ cannot be used as the valuation of leisure time, but some other function $(\theta/\lambda) = \mathfrak{f}(\ldots)$ can still be determined. To simplify his model, Wingo is overly-restrictive.

$$w(d) = g(s + t(d))$$

Since the worker at d would actually be paid a market wage $w_m(d)$ for each of s hours, that market wage may be determined by equating the incomes

$$w(d)(s + t(d)) = w_m(d)s$$

or

$$w_m(d) = (1 + \frac{t(d)}{s}) \cdot g(s + t(d))$$

In the absence of the journey-to-work the market wage needed to elicit s hours of work need only be the "pure wage"

$$w = \quad (s)$$

which is independent of location.

The increment in income necessary to get the workers to overcome the time cost of the journey (in utility dollar equivalents for leisure time cost) then, is

$$V(d) = s(w_m(d) - w)$$

$$= s\left[(1 + \frac{t(d)}{s})\, g(s + t(d)) - g(s)\right]$$

where, since g is presumed to be an increasing function,

$$g(s + t(d)) > g(s) \quad \text{if } t(d) > 0$$

If the dollar travel costs for the journey-to-work are k(d), then the total travel costs c(d) (including time valuation) are, for a worker at d

$$c(d) = v(d) + k(d)$$

The market wage $w_m(d)$ above is that market wage per hour of work required to induce the worker at d to yield s hours of work and t(d) hours of travel for the journey to work. Obviously, $w_m(d)$ varies with the distance (location) d. The plant, however, must pay all workers the same market wage, w_m, and that wage must be adequate to induce the most distant worker to make the journey and work s hours. That is, if that "farthest distance" is $d = \hat{d}$, then the actual market wage w_m paid by the plant must be at least as great as $w_m(\hat{d})$, the necessary market wage for the worker at d. Since it need not be greater,[11] then

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[11] $w_m(\hat{d}) > w_m(d)$ for all $d < \hat{d}$, since $t(d) < t(\hat{d})$ and $g(s + t(d)) < g(s + t(\hat{d}))$.

$$\hat{w}_m = w_m(\hat{d})$$

—the actual market wage equals the acceptable market wage for the "marginal" (most distant) worker.

This wage relationship means that a worker at $d < \hat{d}$ receives an increment in income

$$s(\hat{w}_m - w_m(d)) > 0$$

in excess of what is required to induce him to make his journey-to-work and to work s hours. Moreover, he pays an amount

$$k(\hat{d}) - k(d)$$

less in transportation costs than does the most distant worker. Wingo reasons that this total—the income increment plus the travel cost savings—may be realized by the landowner as an increment in rent that can be charged this worker relative to the rent paid by the marginal worker (at $\hat{d}$). That is, the landowner can increase the rent at d by this amount over that at $\hat{d}$ without forcing the worker here into an untenable position with respect to his income/leisure balance — he is still willing to travel t(d) distance to work s hours for $\hat{w}_m$ wage per hour worked. This increment to rent is called "position rent" and is equal to

$$\begin{aligned} R(d) &= \left[s(\hat{w}_m - w_m(d))\right] + \left[k(\hat{d}) - k(d)\right] \\ &= s\left[(w_m(\hat{d}) - \hat{w}) - (w_m(d) - \hat{w})\right] + \left[k(\hat{d}) - k(d)\right] \\ &= \left[v(\hat{d}) - v(d)\right] + \left[k(\hat{d}) - k(d)\right] \\ &= c(\hat{d}) - c(d) \end{aligned}$$

i.e., the difference in total travel costs, including dollar valuation for the leisure time lost in travel.

Total rent at a location d, then, is the sum of position rent R(d) at d and "location rent" R which is the common component to rent for workers at all locations $d < \hat{d}$. At $\hat{d}$, of course, position rent is zero, and total rent equals location rent. The location rent component for these workers is based on other factors such as supply of land and rents being paid at successive locations by workers at other plants. The resulting rent schedule over distance, when extended over various employment groups, is not unlike the locational stratification resulting from the Alonso bid-rent process modified to include a time budget. It has the additional perspective coupling rent determination to labor market mechanisms. However, by breaking the consumer resource allocation problem into parts via specification of an independent marginal value of leisure function, problems are created which do not come up in the modified Alonso simultaneous determination model. On the other hand, that Alonso model subsumes the Wingo model as a special case.

One obvious problem with the Wingo model is that the schedule of total rents is produced without regard to land consumption for residential purposes. Wingo tries to rectify this by positing an independent demand function for housing in terms of density and unit price, but this begs the question, since that demand function should not be independent of locational choice. Wingo apparently is led into believing that it can be independent because workers in a group are thought to be indifferent among locations under the rent schedule,[12] since that rent schedule assures a proper income/leisure balance for all workers (at whatever locations) according to their (common) marginal value of leisure function.

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[12] See Transportation and Urban Land, p. 65.

It is true that there is some type of equilibrium between income (wage rate and work time plus travel) and leisure for all workers, but there is no assurance that the rent schedule produces a constant level of utility over locations. First, there is ambiguity as to which components of income yield utility, and consequently there is ambiguity as to the nature of time-money equilibrium represented by the marginal value of leisure function. That is, it is not known whether that equilibrium is based on the utility of gross income independent of allocation to uses, income net of rent and travel, or allocations to uses of income separately. Second, there is the fact that the marginal value of leisure function itself, like the labor offer curve from which it is derived, is not an indifference curve. Points on the curve for that function give the wage rate for which a certain amount of work (plus travel time, for Wingo) is forthcoming but give no indication of utility level, which may vary over the curve.

For there to be indifference among locations, then, the rent schedule not only must not jeopardize the desired work levels per worker at all locations but must also produce the same level of utility everywhere. This has strong implications concerning the utility of space consumption and/or rent and travel expenditures, either directly or indirectly through their implications for other expenditures as residuals, since both requirements cannot be met simultaneously without such effects. This brings up the first point, that the marginal value of leisure function is ambiguous on such effects. Indeed, it implies indifference to those expenditure components; otherwise Wingo could not use it as an independent (of location) function. Yet indifference among expenditure components, including rent, would preclude indifference among locations under this rent schedule, since income is everywhere the same, while leisure time varies inversely with distance. Thus, utility differences among locations would make this rent schedule unstable, so that Alonso-type bargaining would lead to new rent levels.

In aggregating behavior through simplification of individual behavior, Wingo creates inconsistencies which would not arise in an individual-oriented model such as Alonso's modified for a time budget. Moreover, the leisure time component in the latter can be detailed to include any of the activity time and frequency perspectives discussed in previous chapters, something not possible in Wingo's model. One approach which uses the time budget version of the Alonso model to try to derive the type of rent schedule that Wingo desires based on travel costs is that of Harris, Tolley and Harrell.[13] They include in the utility function an "amenities" good x_a which is a function of distance but bears no price, and they exclude work time and travel time and cost from the utility function. The dollar value of time thus is equal to the wage rate at equilibrium. With the further assumption that travel time and costs are proportional to distance —

$$t(d) = \alpha d$$

$$k(d) = \beta d$$

— marginal travel costs with time valuation become constant as $(\beta + \alpha w)$. The authors then reason that for reasons of travel a consumer would be willing to give up in rent an extra amount

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[13] R. N. S. Harris, G. . Tolley, and C. Harrell, "The Residence Site Choice," Review of Economics and Statistics, Vol. 50, May 1968.

$$(d - \hat{d})(\beta + w\alpha)$$

to move from location $\hat{d}$ to a closer location d, where $d < \hat{d}$. As for Wingo, this rent increment is the sum of travel cost savings and the dollar value of travel time savings, although here work time may vary. To express this as a bid price curve per unit housing, relative to some location d and the rent there, the authors divide the increment by rx_h, where r is the capitalization rate and x_h the housing purchased. This curve is, after all, a particular form of the general case considered previously.

Real world deviations from the travel-based curve derived above were thought by the authors to provide an estimate of the amenity effects omitted from that curve. These they try to identify from the location equation for their model — which is

$$\frac{\partial \mathcal{U}}{\partial x_a} \cdot \frac{\partial x_a}{\partial d} - \lambda x_h \left(\frac{\partial p_h}{\partial d}\right) = \lambda \left(\frac{\partial k_d}{\partial d}\right) + \theta \left(\frac{\partial t_d}{\partial d}\right)$$

— rearranged and modified as above to become

$$\frac{1}{\lambda x_h} \frac{\partial \mathcal{U}}{\partial x_a} \frac{\partial x_a}{\partial d} = \frac{\partial \hat{\varphi}}{\partial d} + \frac{1}{x_h}(s + w\alpha)$$

where $\hat{\varphi}$ is a statistical estimate of the market rent function. Thus their measure of amenity effects is an aggregate measure.

It is evident that if amenities yield positive marginal, they must increase with distance if the right-hand side is positive and decrease with distance if it is negative. There does seem to be some circularity here, for amenity may be more exactly a function of the resulting settlement pattern than of distance per se, so that the observed pattern need not represent an equilibrium

Activity Pattern, Residential Location and Space Use

Residential space, or housing or land, may be identified as one of the goods – or even as a set of those goods, if differentiated by type or component to a residential "bundle" – used as inputs to activities in the variable proportions or in the fixed proportions (Becker-DeVany-Lancaster) models of the previous chapters. As residential location changes, the price(s) of the residential good(s) can be expected to change, resulting in changes in the use of goods of all kinds, including residential, in activities. Other dimensions of activities, such as time and frequency, will change with changes in goods allocations, and also with changes in unit time and money costs of travel as residential location changes. Residential location change, then, means change in relative accessibility to opportunity for activity and leads to readjustment of the entire activity pattern. Finally, there may be location-specific amenity or "environmental" variables which impute a utility to a location over and above its accessibility effects for allocations from limited budgets of time and money.

As became apparent in the discussion of Isard's location model in terms of shopping pattern, when activities (or opportunities for them) are distributed over space rather than along a line, determination of an optimal residential location—and consequently the optimal activity pattern and space use—becomes extremely difficult even in theory, so that it may not be possible to characterize the nature of such optimal solutions (in terms of first order conditions) in sufficient detail even for didactic purposes. This is not to mention the fact that in the real world residential "bundles" tend to be discrete, location-specific entities distributed over a plane that is far from uniform.[14] Wilson has subjected consumer optimization to additional "space budget" and other inter-activity and space-time constraints in an activity choice model in time and money in an attempt to recognize (in continuous fashion) some of the limits to freedom in residential activity allocations.[15] However, these do not serve to clarify the nature of optimal solutions to such problems.

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[14] See J. Herbert and B. H. Stevens, "A Model for the Distribution of Residential Activity in Urban Areas," Journal of Regional Science, Vol. 2, 1960, pp. 21-36, for consideration of discrete residential choice. Their model was a provocative input to model development in the Penn-Jersey Transportation Study. Britton Harris has pursued many of the issues raised in that study in his own research. See, for example, Britton Harris, et. al., "Research on An Equilibrium Model of Metropolitan Housing and Locational Choice: Interim Report," (Philadelphia: Institute for Environmental Studies, University of Pennsylvania), March 1966.

[15] A. G. Wilson, "Some Recent Developments in Micro-Economic Approaches to Modelling Household Behavior, With Special Reference to Spatio-Temporal Organization," in his Papers in Urban and Regional Analysis, (London: Pion Limited), 1972, pp. 216-236. His review covers some of the developments cited in these chapters, but he does not treat activity frequency and its travel implications. He does stress aggregation problems in optimizing models of consumer behavior and has developed a probabilistic approach.

By way of illustration, we can derive the location equations for the set of location variables l_j in the variable proportions model with time (t_k), frequency (f_k) and goods (x_{ik}, $i = 1, \ldots, n$) inputs to activities $k = 1, \ldots$. Assuming a single residential good $i = h$, the first order condition for the j^{th} location variable (e.g., a coordinate), $j = 1, \ldots,$ is

$$\frac{\partial \mathcal{U}}{\partial l_j} - \lambda \left\{ \frac{\partial p_h}{\partial l_j} \left(\sum_k x_{hk} \right) + \sum_k f_k \left(\frac{\partial \theta_k}{\partial l_j} \right) \right\}$$

$$- \theta \sum_k f_k \left(\frac{\partial \alpha_k}{\partial l_j} \right) = 0$$

where the activity frequency f_k is also the number of trips made for that activity at cost θ_k per trip and taking α_k time per trip, the latter both being functions of location. Such conditions tell us that at the optimal location the marginal utility with a location variable just equals the full marginal costs — through trip pattern for activities and housing inputs — with that location variable. (For work to be included among the k, the appropriate cost function s could be corrected for the shift wage.) Information on the nature of the optimal location is minimal without specification of the environment of choice, of course. These conditions are analogous to those in the Isard model, with the addition of the housing costs and a utility to location.

Becker has worked out an example for the fixed proportions activity model. First, let us consider a more general statement than his example and relate it to the variable proportions case by defining x_{tk} as the amount of travel good i = t used in activity k, where this can be assumed to be proportional to the number of trips made:

$$x_{tk} = \gamma_k f_k$$

As for all goods in the Becker model, this good is proportional to activity level —

$$x_{tk} = c_{tk} A_k$$

(meaning that the utility of A_k must include travel effects, if any) — so that

$$f_k = \left(\frac{c_{tk}}{\gamma_k}\right) A_k$$

Further, if travel time and cost per trip are α_k and β_k, respectively, then the time and cost per unit good t are (α_k/γ_k) and $\beta_k/\gamma_k)$. This means that the time input coefficient b_k for activity k covers activity time only, to which travel time must be added. With these modifications the first order conditions (the location equations) become

$$\frac{\partial \mathcal{U}}{\partial l_j} - \sum_k \left\{ \lambda (c_{hk} \frac{\partial p_h}{\partial l_j} + \frac{c_{tk}}{\gamma_k} \frac{\partial \beta_k}{\partial l_j} + \theta \left(\frac{c_{tk}}{\gamma_k} \frac{\partial \alpha_k}{\partial l_j} \right) \right\} A_k = 0$$

where inputs of housing to activities are given through c_{hk}.

Becker simplifies the problem by taking housing and travel out of the proportionality relationships and posing the Alonso problem of location along a line, with no frequency considerations and trips aggregated into the general time and cost functions t(d) and k(d) with distance d from the center as the argument. As a labor-leisure problem, the budgets are

$$T - t_w - \sum b_k A_k - t(d) \geq 0$$

$$wt_w - \sum_k \sum_{i \neq h} p_i c_{ik} A_k - p(d)x_h - k(d) \geq 0$$

where b_k gives activity time per unit and c_{ik} gives input of good i to activity k per unit, good prices being the p_i. By postulating neutrality toward work, he eliminates work and one budget by substitution for t_w. The resulting budget is

$$wT - h(A) - p(d)x_h - k(d) - wt(d) \geq 0$$

where Becker denotes "total consumption" as $h(A)$. It is easy enough to show that

$$h(A) = \sum_k (wb_k + \sum_i p_i c_{ik}) A_k = wt_l + x_m$$

where t_l is non-travel leisure time and x_m is "other expenditures" as in Alonso's model. Quite naturally, then, his location equation —

$$\frac{\partial p(d)}{\partial d} = - \frac{1}{x_h} \left\{ \frac{\partial k(d)}{\partial d} + w \frac{\partial t(d)}{\partial d} \right\}$$

— is that of the modified Alonso model (and the Harris-Tolley-Harrell model) with $w = (\theta/\lambda)$ due to the neutrality of work.

No further attempt will be made to detail location equations for various specifications of activity models, for the reasons given previously. It is hoped that this discussion will provide some background to location problems which can serve as a frame of reference in reviewing location-activity relationships in the River Basin Model. The rather extensive review of Wingo's model was undertaken precisely because its reliance on dollar valuation of time in location/journey-to-work/rent relationships is not unlike the method of the River Basin Model.

SECTION VI
SUMMARY

That the consumer optimization problem can be made as comprehensive -- and complex -- as desired should be evident from the diversity of behavioral situations reflected in the models presented in these chapters. It should also be evident that such complexity can be at the expense of clarity and operational feasibility. Indeed, the statements and derivations as presented here give only minimal information about the solutions to the problems they describe. It is true that the generality of the first-order optimization conditions is a positive factor, in that it leads to rules-of-thumb concerning relations "at the margin" which should hold for all utility functions of the types postulated. But to say anything of significance about actual patterns of trip and activity frequency and time and money allocations requires going beyond the general cases here to specification of the utility function itself. Choice of that function often seems far too arbitrary in the literature. Selection of a specific utility function should only follow the careful detailing of postulates about behavior, whereas behavioral impli-

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cations of different functional forms tend to be treated in rather cavalier fashion in most applications.

The models described heretofore, then, sketch the range of behaviors relevant to River Basin Model considerations. They illustrate how the general principle of consumer optimization -- that a course of action should be pursued until its marginal utility equals its marginal cost in market and resource (shadow) prices -- would be manifest in various choice situations. Simulation in the River Basin Model is to involve incremental choice-making under imperfect knowledge, rather than the perfect knowledge simultaneous determination of these consumer optimization models. It is also aggregated, with "population units" in place of individuals. It has been argued that in the River Basin Model distinction as to scale or size of the behaving unit in the social sector is trivial. Therefore, even without global optimization the "marginal utility equals marginal cost" principles of the consumer models can be used as rules-of-thumb to suggest the direction and relative extent or proportions of incremental changes to budget allocations that the consumer (or population unit) might make in response to outcomes of previous allocations. These rules can be framed in the language of the consumer models, that is, in terms of marginal rates of substitution and relative values of resources (i.e., the dollar value of time).

As it now stands, the River Basin Model does indeed view Social Sector (population unit) behavior in much this way, although in a gaming context. In complete simulation of the Social Sector, decision-maker input procedures must be replaced with mechanisms for internal determination of incremental population unit allocations. These mechanisms might well be based on just these rules-of-thumb from the utility-maximizing models of consumer behavior. It is not the charge of this research to develop these simulation procedures, although there will be occasion to comment on their development later. It is hoped that this review will suggest behavior components to such mechanisms. At this point, we will turn to a direct critique of the River Basin Model, where the current structure of the model will be reviewed in terms of the consumer theory and other research.

PART TWO

THE RIVER BASIN MODEL

SECTION VII
THE STRUCTURE OF THE MODEL

In the chapters to follow the operation of the River Basin Model as it affects the population units of the Social Sector will be thoroughly reviewed. Source material for this review are the program listings of March 11, 1972, called SOURCE, and the NEWLIB update of March 6, 1972, plus the "River Basin Model" reports in the U.S. Environmental Protection Agency's Water Pollution Control Research Series prepared by Environmetrics, Inc., and listed as Project # 16110 FRU, Volumes 1 through 12.

In this critique, every attempt will be made to adhere to the original subprogram names, variable identifications, and programming formats from the source listings to facilitate cross reference. As there are between three and four hundred subprograms, however, some liberties will be taken in the description and analysis, so that functions may be attributed to managing subprograms instead of to the computational subprograms they call for actual system operations. Similarly, since the objective here is the review of the model's operation as it affects the Social Sector, not program debugging, there may be technical accuracies which slip through in this presentation. These should not be of real consequence for our purposes.

An Overview of Model Operation

In this chapter the structure of the River Basin Model will be outlined. The purpose here is to provide a sketch of the operating modules of the model and the manner in which they tie together. In subsequent chapters, those modules (or groups of subprograms) most relevant to population unit behavior will be reviewed in detail. The behaving "sectors" of the model are presented here in their gaming format, as originally specified in the model. However, the move to complete simulation of a sector can be accomplished by replacing the decision-maker input procedures for that sector each round (section "A" in Table 1, below) with subroutines which adjust sector demand and operating parameters in response to previous round outputs according to behavioral rules for sector choice-making (as, for example, from rules-of-thumb extrapolated from the consumer theory to population unit behavior in the Social Sector). From that point on, the model can operate much as it does at present.

Model structure, in terms of major subprograms, is outlined in Table 1 below. An attempt has been to separate the managing subprograms (in the left-hand column) from the major operating subprograms they call (in the center and right-hand columns). These operating subprograms are numbered opposite the calling subprogram in the order they are called. The actual subprogram name is given in parentheses following a simple sectoral or functional identification for that subprogram. The managing subprograms are themselves listed in the order in which they are called (A through M) by the main program, CTYMAIN. Operating subprograms are separated into columns according to whether or not they directly determine Social Sector behavior. Those subprograms jointly determining social and other sector behavior are listed between the center and right-hand columns, with dashes to either side to indicate this common relevance. Those subprograms of special import to the Social Sector and thus falling in the center column will be considered in detail below and in the following chapters, usually grouped to have a common calling subprogram. Although the water system is a focal point in the River Basin Model, since population units have so little direct say over water-related matters there will be little reference in this critique to the water system outside of its role in the determination of the level of the Environmental Index for population units.

The general procedure for a round's operation of the model is fairly obvious from Table 1. First, each sector specifies the desired levels of operation of activities over which it has control (A). For the Social Sector this means the desired work and consumption patterns for population units, including time and money allocations and relationships with institutions, such as political and organization ties. These desired levels for the round can be viewed as extensions or adjustments to previous desires (or "demands") in light of the degree of "success" realized, however success is defined by the behaving units of the sectors. Next, some population units are redistributed by location within the system, partly in response to dissatisfaction with environment and activity manifestations (B). After certain adjustments to the system (C and D), employment relationships are re-

TABLE 1: Sequence of Operations in a Round of the River Basin Model (via CTYMAIN)

Program Management	Social Sector Activities	Activities of Other Sectors
A. System up-date from input decisions (SETEDT, EDTMAIN, EDIT)	5. time allocations (TALOC) 6. dollar value of time (TMVAL) 7. vote (CASTER)	1. land purchase (PU) 2. land use alterations (BUILD) 3. financial operations (NCHPVT, SCHPVT) 4. utilities (CHGUTS)
	- - - - 8. cash transfers (CASHT) - - - -	
	10. summary(TIMET)	9. summary (LOSTT, SPPTET)
	14. boycotts (BUYCT)	11. rail system (RAIL) 12. tax rates (TAXEZ) 13. assessment (AS) 14. redistricting (REDIST) 16. water, facilities (WRPRC, WRBLD)
	- - - - 17. external inputs (ODDS, ENDS) - - - -	
		18. bus routes (ADDBUS)
	- - - - 19. summary - - - -	
B. Migration (MIGMAIN)	1. dissatisfaction, migration (MIGRAT)	
C. Water System (GAILMAI)		1. water system (SETSTF, GAILMN)

TABLE 1 - Continued

Program Management	Social Sector Activities	Activities of Other Sectors
D. Post-Demolition Operations (KLEAR)	1. educational stratification (EDORD)	2. depreciation, value ratios (DEPREC) 3. assessed value (ASVSET) 4. job opportunities (SETEMP)
E. Employment and Transportation Assignments (EMPMAIN, TRCMAIN)	- - - - 5. full time employment and transportation (EMP, TRTRC)	- - - -
	- - - - 6. part-time employment and transportation (EMPRT)	- - - -
		7. business capacities (SETCAP)
F. System Loads and Further Allocations (ALCMAIN)	- - - - 1. employment summary (EMPSUM)	- - - -
	5. adult education (NSPACK)	2. municipal service loads (LOADMS)
	6. recreation and park use; voter registration and education level (TMALC)	3. municipal service quality (MSQUAL) 4. school loads (LOADSC)
G. Commercial Transactions		1. construction contracts (CONAC) 2. prices, costs, taxes (PRCSET)

TABLE 1 - Continued

Program Management	Social Sector Activities	Activities of Other Sectors
	- - - - 3. demand for goods (SECTOM)	- - - -
	- - - - 4. shopping assignments (OPCM)	- - - -
	- - - - 5. summary (COMDIG)	- - - -
H. Terminals (TRMMAIN)		1. terminals (TERMS)
I. Output for Private Sectors (PRYMAN)	1. social (WRYOU)	2. other (miscellaneous)
	- - - - 3. summary (PRINTY)	- - - -
J. Water System Output (WATRMAI)		1. water system (WATOUT)
K. Public Departmental Output (PUBMAIN)		1. utilities (UTS) 2. municipal services (PWS) 3. planning and zoning (PZ) 4. school (S[illegible]OUT)
L. Other Governmental Output (BSHMAIN)		1. chairman (CHFO) 2. highway (HYWAY) 3. bus and rail (BSRROT)
M. Summary Statistics (IDMAIN)		1. government (GOVMNT)
	- - - - 2. demographic and economic (IDEMEC)	- - - -

considered (E). Workers in population units may seek new or better jobs or be forced from previous ones according to their ability to complete (education), their willingness to travel (time versus money), and changes in the level and distribution of jobs available. The new employment relationships result in new transportation loads and institutional operating capacities. Under these new capacity constraints, population units attempt to realize their demands for facility use in terms of time (F) and money (G), while institutions make similar transactions among themselves in keeping with their demands and system capacities. Finally, the impacts of these actions on the system are assessed and summarized (H through M).

At the end of this sequence for a round, then, the system is in a new "state," and the behaving units' experience has been determined accordingly. The model is thus ready for a new set of (incremental) demands to initiate another round. The sequence of outcomes to behaving units over time traces the behavior of the system as a whole and in all its parts. Below, the nature of Social Sector inputs (allocations) for a round will be explained and commented upon. Successive chapters will take up migration, employment, and consumption pattern.

Social Sector Inputs and Allocations

The "inputs" for population units of the Social Sector each round are the choice variables open to those units. Therefore, the specification of these inputs defines the behavioral alternatives open to the (partial) control of the behaving unit, and this is true whether decisions concerning input levels are made externally (gaming) or internally (machine simulation). There are five sets of choice variables for population units here and five subprograms for entering them: allocation of leisure time (TALOC), setting of a dollar value to time in transportation (TMVAL), casting votes for political offices (CASTER), transferring cash from savings to other sector accounts (CASHT), and instigating boycotts of facilities or institutions (BUYCT).

Before discussing these, let us make explicit some choices which are not open to population units in this model, using as a frame of reference the optimizing models considered earlier. First, and most important, population units are not allowd to allocate money to uses independent of time allocations. Indeed, money expenditures are largely proportional to time expenditures, the only tradeoff provided between the two coming in the role of dollar valuation of time in travel to be explained later. From the viewpoint of pure theory, this would lead to situations where there is a great likelihood of a slack constraint and a zero shadow price to time or money, meaning either a zero or infinite valuation of time in dollars. As it turns out, there is no money budget constraint at all, nor any loss of utility to dissaving. Under such circumstances in pure theory there can be no shadow price to money, and the shadow price of time is in terms of utility equivalents. These are issues which will be treated later in greater detail. The dictates of the pure theory are not the final standards for judgment, of course, especially when there are so many deviations from the pure choice situation. Still, these issues force careful consideration of the type of behavior that is being implied in the specification of the problem and in the definition of the choice or instrumental variables.

Other choices not open to population units here but receiving attention in the consumer theory as it has been developed are activity frequencies, including numbers of trips, and residential location and space use. As in many non-frequency optimizing models, one might suppose that fixed proportionalities for activity time and frequency (in terms of a given time per episode) and for activity cost and frequency (in terms of goods inputs or admission prices or fares per episodes) lie at the heart of postulated proportionalities between time and money expenditures, but the model is not explicit in this regard. Space use is considered a function of the type of population unit, based on income, and is held fixed for a population unit whereever it resides. Interestingly, the classification of the population unit never changes — thus neither does its use of housing — even though its income and education can vary (only within strict limits, it is true, except in the cases of under-employment and unemployment). Migration from a residential location is partly probabilistic and partly behavioral, as is the choice of the new residential location. However, the behavioral rules involved are divorced from other consumer choices, unlike the optimizing model, where they interrelated. In the River Basin Model, residential location does have implications for other choices and outcomes, of course. Again, these are all issues which will come up in the review of the appropriate operating subprograms.

In TALOC, then, each population unit specifies the times it desires to allocate to each of five leisure time activities. This information is stored in the vector

TIME (K, 1)

where the index K gives the uses

1 = part time work
2 = public education time (adult)
3 = private education time (adult)
4 = political activity time
5 = recreation time

The index "1" indicates that these values are times requested; thus these time allocations are demands. At the end of a round there will be output another vector, TIME (K, 2), giving the actual times in these activities that the population unit was able to realize. These TIME (K, 2) values are thus time expenditures as differentiated from the allocations, where the model will assign times such that

$$\text{TIME } (K, 2) \leq \text{TIME } (K, 1)$$

— that is, expenditures will not exceed allocations. They will be less than allocations if there is some reason, due to the behavior of the system, that the population unit cannot achieve its objectives. Thus they will be as close to the allocations as the operation of the system will allow. TIME (K, J), then, is a matrix of times for allocations (demands) and expenditures (outcomes). There is also a K = 6 level, which gives travel time to work for J = 1, not a choice variable, and involuntary time (defined below) for J = 2. Again, TALOC simply inputs the choice values for $K \leq 5$, J = 1. There are subscripts for the class and location of the population unit, but these will be suppressed in this review for simplicity unless needed.

Both time allocations and time expenditures are subject to a time budget constraint whereby each adult in the population unit has 100 time units to devote to leisure activity. All adults in the population unit (composed of 500 persons) are presumed to allocate their leisure time identically.[1] Regular job work time is 80 time units per worker and is not included in the (leisure) time budget. It might be noted that although the labor force participation rate for a population unit — and thus its total time at work — varies by class of population unit, the stock of leisure time does not change. This seems a little strange, for it means that some adults have less total time than others. If allocation times do not total in excess of 100 units per adult, neither will time expenditures for those activities ($K \leq 5$). However, expenditures must include two non-allocational items, illness and travel time, both of which are determined in activity assignment procedures based on system capacities. These are deducted from the leisure time stock of 100 units before other time expenditures are determined (see TMALC).

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[1] This is the prime justification in viewing population unit behavior as the behavior of an individual and using the optimizing model of utility theory as a frame of reference. Strictly speaking, not only all adults in a population unit but also all population units of the same class on a parcel (residential location) have the same leisure time allocation.

Therefore, the residual time stock for leisure activity time expenditures may be considerably less than 100 units, and there may not be enough time to meet all allocations even if the system has the capacity to permit them. On the other hand, if expenditures are so limited by system capacities, or if a significant amount of time were not allocated, the sum of time expenditures, plus illness and travel, may leave some time stock unused. Such slack time is "involuntary time,"[2] TIME (6, 2), although failure to allocate enough time would certainly appear to be voluntary. Such a failure is not likely to happen without a money budget constraint, however.

The relative sizes of regular work and leisure time stocks — 80 and 100 units, respectively — means that of the total 180 time units, the working adult spends 44% in his full time job. This does not appear to be too different from the real world situation. In Washington, D.C., for example, an adult working a 40-hour work week can be expected to have about 34 hours in the leisure categories K = 1, ... , 5 above, plus about 5.5 hours in the journey-to-work, per week.[3] Of the total, then, almost exactly 50% is work. Sleeping, eating, childcare and other routine activities are not included. By giving working and non-working adults alike the same leisure time stock, the River Basin Model implies that the non-workers match the job time of the workers with additional household and childcare activities, but non-workers in the Washington study had well over ten more hours more in leisure each week than workers had. Moreover, "lower" class population units not only have higher labor force participation rates but also higher birth rates (number of children) per population unit, meaning that "higher" class population units must either have more leisure or else be spending an inordinate amount of time in housework and childcare, whereas they are the most able to hire help. Such implications and possible inconsistencies as these should be analyzed in detail and adjusted to fit real-world patterns before planning simulations are made based on time allocation procedures. Elsewhere in this critique many other suggestions will be made for bringing the River Basin Model time budget more in line with reality.

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[2] This is the way involuntary time is computed in TMALC (see below). Manual # 4, p. 90 states that involuntary time equals the difference between allocated and expended times in extra job and education acticities. The TMALC method can yield zero involuntary time when the manual method does not, and vice versa.

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[3] The metropolitan Washington, D.C. time budget study will be used as a frequent reference. See Hammer and Chapin, op. cit.

Subprogram TMVAL simply inputs the dollar value the population unit attaches to a unit of time in travel, and TIMET prints out a record of these inputs. This parameter serves as a shadow price on travel time; increasing the parameter increases the chances that the system will assign the population unit to a faster but more expensive transportation route/mode alternative for the journey-to-work, and decreasing it biases assignment toward a slower but cheaper alternative. The parameter, then, represents a preference in time-money substitutions, and in theory it should be consistent with other preferences for time uses and with relative prices or costs of other activities. At present there are no such consistency requirements, and we will be especially interested in seeing what kinds of anomalies can arise. Since the objective function is the same for all population units, if each tries to maximize satisfaction differences in the dollar value of time parameter should directly reflect differences in prices and costs — travel and non-travel — by residential location.[4] Actual transportation costs paid do not include this shadow price, of course. By manipulating the parameter the population unit attempts to minimize divergence of the activity time expenditures it realizes from the activity demand allocations it chose in presumably attempting to maximize satisfaction. In complete simulation the parameter should be a simulataneously determined index rather than an independent choice variable.

The only direct money manipulation open to the population unit is the transfer of cash from savings to the accounts of other sectors, as personal loans or contributions. There is no direct allocation of money by the population unit for household consumption expenditures; instead, current personal expenditures to cover transportation, health, education, taxes, and consumer goods and services are determined in population unit activity time and location assignments, so that time expenditures represent money expenditures as well. The only constraint on cash transfers by the population unit is that it not exceed savings. Population unit cash transfer decisions are input in CASHT, and these may have to be limited in complete simulation of the Social Sector because they are so closely tied to over-the-table bargaining. Otherwise, we may end up with mechanistic vote- or political favor-buying.

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[4] Similarly, in theory these price and cost differences are the only reasons for differences in time allocations (demands). See discussion of the objective function in MIGRAT (HSDSSW).

Voting and boycotting are the final two choice-areas open to the population unit. CASTER enters population votes for political officers, presumably cast in reaction to their previous behavior and current policies. Before voting can be completely automated, some index of the salience of a candidate to the population unit in terms of that behavior and those policies must be constructed. The total number of votes which can be cast by a population unit is limited to its voter registration level as a function of time spent in political activity in the previous round (see TMALC).

While time allocations are in effect labor supply and consumer demand statements without specific reference to source of employment or supply or to spatial wage or price differences, boycotts permit population units to exert pressure for change in the operation or behavior of particular units in other sectors by intentionally constraining the opportunity choice-set to be considered by employment or other activity spatial-temporal system assignment procedures. Units boycotting identify the units to be boycotted in the subprogram BUYCT, and those boycotted units will not be included among opportunities for activity assignments for the boycotting units.

There are already certain built-in preferences or restrictions on the behavior of population units by class which amount to class-wide permanent boycotts of other types of behaving units. For example, low class population units cannot use private schools, and high class ones cannot (will not) live with low class ones. The second case here appears to be a behavioral "social boycott," but the first is ambiguous, as it could mean either discrimination by private schools or financial inability by population units. The latter would best be handled by introducing a money budget constraint for population units and then allowing low income population units to lobby for price changes in education as normal political (voting) or boycotting behavior. These examples point out the need to separate behavioral propensities of behaving units from functional constraints on their behavior. Functional constraints which could be relaxed through changes in the system should not be confused with behavioral propensities (preferences) and thus permanently built into system simulations. This issue will come up several times in this critique. It suggests that as many of these propensities as possible be made characteristics of preferences (objective functions) differentiated by class rather than presented as "permanent boycotts."

As with voting, boycotting may be hard to simulate internally because of its bargaining aspects, it being a reasonable course of action for a population unit only if others can be convinced to do likewise. Otherwise, the opportunity costs in time and money to the boycotting unit may be too high with little chance of success. In simulation threshhold levels for instigating boycotts would have to be established as functions of relative price levels and/or service quality levels if boycotting is to be retained and carried out automatically. Population units which are unsuccessful in boycotting would be prone to leave the system, so that boycotting could be viewed as a last-ditch effort before outmigration occurs for dissatisfied population units. Widespread dissatisfaction and outmigration would be dampened because increased boycotting would improve chances for system change (see MIGRAT). Voting behavior could have the same effect. Again, there would have to be mechanisms for identifying the responses appropriate for desired changes in the system, degree of success being solely a matter of the number of population units with similar enough experience to evoke the same response.

This discussion of current input procedures has set up the population unit choice problem and raised some issues relevant to the machine simulation of this choice-making. We will turn in the next several chapters to system assignment procedures. These procedures in most cases can be operative under a wide range of allocational simulation procedures, since they require from population units only the choice inputs described here, however derived. Similarly, there are some problems with these procedures which would be evident regardless of the nature of input derivation. Finally, there are some instances in which the nature of the choice-making simulation interacts with the assignment operation, so that modification of the latter is dependent on the selection of the former. These aspects will be drawn together in the concluding chapter to the critique.

SECTION VIII
DISSATISFACTION AND MIGRATION

The River Basin Model contains an explicit "objective function" for the population units of the Social Sector, and all population units are thought to share the preferences represented by this function. In this case, not only do all population units have the same preferences but also view utility or satisfaction with a scale and origin to the function which is common to all, so that interpersonal (inter-population unit) comparisons of utility level are possible. Indeed, such comparisons are actually used in the migration procedures to screen out the "most dissatisfied" population units for migration purposes.

This chapter will discuss the objective function, which it is presumed that population units set levels to their choice variables (including time allocation) to optimize, and the migration procedures, which utilize the level of the objective function as one among several factors in migration behavior. The level of the objective function, of course, gives the "value" of the outcomes of a round's operation of the system to the population unit in terms of the satisfaction of

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experience that round. Migration is the only direct consequence the population unit might have to face purely because of its satisfaction level, and as such it is an extension of its choice behavior. That is, the decision to migrate is a part of normative population unit behavior that is simulated internally in the model.[1] Following chapters will examine just how the outcomes to the population unit are determined through activity assignment procedures.

The Quality of Life Index

The objective function is called the Quality of Life Index because it has as arguments measures of environmental quality, of which manifest life style in terms of time expenditures is a component. It is also called the "dissatisfaction index" because it actually measures increasing levels of dissatisfaction.

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[1] Perhaps the behavior is more truly descriptive than normative, in that a certain percentage of the most dissatisfied simply tend to migrate. It would be normative, like allocational decisions, if a population unit would be better off to migrate when its satisfaction reached a certain level.

Thus, optimizing behavior would be to select choice variables hopefully leading to outcomes which would minimize the level of the index. In reviewing the composition of the index it will become apparent that most arguments to the function for an individual are beyond the direct control of the population unit. That is, while time allocations have a direct role in determining time expenditure outcomes, other components of the index can only be influenced indirectly by the population unit in its political (voting) or boycotting behavior.

That time expenditures are only a few of the many components in the Quality of Life Index, as computed for population units in subprogram HSDSSW,[2] can be seen in its structure as the sum of four separate indices: the Pollution Index and the Neighborhood Index (together, the Environmental Index) and the Health Index and the Time Index (together, the Personal Index). These indices are given equal weight, but only the Time Index has as arguments actual time units. The others are functions of quality and service indices which do reflect population unit time and money expenditures, but for whole classes of users in addition to the population unit for which the index level is being computed.

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[2] Or HSDSST, in the event that the water system is not included.

Because these other indices for a population unit depend on the activities of other units, and because control can be exercised only politically or through boycott in concert with other population units, then if the Time Index is not sufficiently weighted relative to those others in the Quality of Life Index, time allocations and resulting expenditures for a population unit may fluctuate wildly from round to round in response to small changes in system prices and parameters but having very little direct impact on the dissatisfaction level for that population unit. Such time fluctuations lead to fluctuations in consumer demand and then to oscillations throughout the economy and the system. Population unit control over its destiny must be adequate to stabilize its behavior. Otherwise, the time budget approach is best abandoned. Because voting and boycotting behavior may be difficult to simulate internally, they cannot be counted on as sources of that control.

In order to assess the sensitivity of the Quality of Life Index to time expenditures, and thus tentatively to time allocations on which they are based, the component indices will be defined below without much comment on non-time arguments, the Quality of Life Index being the sum of the values of the component indices for a population unit.

1. PLIN: Pollution Index for a parcel "I"

$$\text{PLIN} = \begin{cases} (\text{WTQL}(I) - 3.5)^3 & \text{if the parcel has surface water} \\ \frac{1}{2}\left(\frac{1}{N}\right)\sum_{J}^{N} (\text{WTQL}(J) - 3.5)^3 & \text{if the parcel does not have surface water, where } J = 1, \ldots, N \text{ denotes adjacent parcels} \end{cases}$$

where WTQL(J) is the water quality rating of parcel J (or I),

$$\text{WTQL}(J) = \underset{\text{over } K}{\text{maximum}} \left(\text{POLN}(J, K) \right)$$

where POLN(J, K) is the quality level for pollutant of type K on parcel J, K = 1, . . . , 7 (including coliform bacteria)

$$0 \leq \text{POLN}(J, K) \leq 9$$

thus

$$-(3.5)^3 \leq \text{PLIN} \leq (5.5)^3$$

2. HD: Neighborhood Index for a population unit of class "K" on a parcel "I" in political jurisdiction "J"

HD = CTYDST(1) + CTYDST(2) + RNF + QIF +
    + JURFAC (J, K)

where

2.a. CTYDST(1): Municipal Services Quality Index

CTYDST(1) = PUBFAC(1, K)* AMAX1(0.,
    PUBVAL(1, I) - PUBMIN(1, K))

where PUBVAL(1, I) $\le$ 200 is the municipal services level (100 times the ratio of demand to supply capacity), PUBMIN(1, K) is the acceptable utilization level, and the difference is overutilization; AMAX1 takes the maximum of zero and overutilization; PUBFAC(1) is a scale factor

2.b. CTYDST(2): School Quality Index

CTYDST(2) = PUBFAC(2, K)* AMAX1(0.,
    PUBVAL(2, I) - PUBMIN(2, K))

where the arguments are analogous to those above for municipal services

current data:

for all L = 1, 2 and K = 1, 2, 3

PUBFAC(L, K) = 1
PUBMIN(L, K) = 100

therefore, for L = 1, 2

0 $\le$ CTYDST(L) $\le$ 100

(See subprogram DEPREC for computation of supply capacities.)

2.c. RNF: Rent Charged Index for a population unit of class "K" on parcel "I"

RNF = RNTFAC(K)* AMAX1 (0., RENT(I) - RNTMIN(K))

where RENT(I) is the rent charged per space unit, RNTMIN(K) is a "typical" rent per space unit for this class, and AMAX1 takes the maximum of zero and the "excess rent" charged; RNTFAC(K) is a scale factor for class "sensitivity" to excessive rent

current data:[3]

$\text{RENT(I)} \leq 210$

K =	1	2	3
RNTFAC(K)	3	2	1
RNTMIN(K)	140	150	165

therefore

$0 \leq \text{RNF} \leq (210, 120, 45)$

2.d. QIF: Residential Quality Index for a population unit of class "K" on parcel "I"

QIF = QIFAC(K)* AMAX1(0., QIMAX(K) - QI)

where QIMAX(K) is the lowest quality index (lowest quality) the unit would accept on moving into new housing, QIFAC(K) is a scale factor, and QI is the residential quality,

QI = MAX0(QUAL(I), MNQI(I))

where MNQI is the maintenance level set

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[3] These data are from manual #3, p. 130. HSDSSW gives RNTMIN(K) = (135, 150, 160).

by the owner (if MNQI > QUAL, then property is being updated), and QUAL is the condition of the property, being a function of age, municipal services, flood and fire factors (see subprogram DEPREC; if MNQI < QUAL, the property is being allowed to deteriorate; MAXO picks the maximum of maintenance and quality levels;

AMAX1 picks the maximum of zero and the deficiency in quality of the residence

current data:

K = 1			
QIFAC(K)	0.5	0.5	0.5
QIMAX(K)	140	180	200

$0 \leq \text{MNQI}(I) \leq 200$
$0 \leq \text{QUAL}(I) \leq 200$

therefore

$0 \leq \text{QIF} \leq (70, 90, 100)$

2.e. JURFAC(J, K): Tax/Welfare Index for a population unit of class "K" on parcel "I" in jurisdiction "J"

If K = 1,

$$\text{JURFAC}(J, 1) = \text{WLFAC* AMAX1}(0., \text{WLFMAX} - \text{PCSH}(30, J))$$

where WLFMAX is the minimal acceptable unemployment compensation rate and PCSH(30, J) is that rate for the jurisdiction, AMAX1 takes the maximum of 0 and the rate discrepancy, and WLFAC is a scale factor

If K > 1,

$$\text{JURFAC}(J, K) \sum_{L=1}^{2} (\text{TAXFAC}(L, L)^{*}$$

AMAX1(0., TAXS(L, J) - TAXMIN(L, K)))

where TAXS give the tax rates in mils for land and development (L = 1) and income and consumption (L = 2), TAXMIN·(L, K) are the respective maximal "acceptable" tax rates by class, AMAX1 takes the maximum of zero and the excess tax rates, and TAXFAC are scale factors

current data:

WLFAC = 4.0; WLFMAX = 20
TAXFAC(L, K) = (.125, .250) by L = 1, 2, for all K
TAXMIN(L, K) = 0.0, all L, K

for K = 1, JURFAC(J, 1) $\le$ 80
for K > 2, for example, if the rates were 20%, then JURFAC(J, K) = 75; obviously, TAXS(L, J) $\le$ 1000, L = 1, 2, so that JURFAC(J, K) $\le$ 375

2.f. Summary:

$$0 \le HD \le (855, 785, 720) \text{ for } K = 1, 2, 3$$

3. HLIN: Health Index for a population unit of class "K" on parcel "I"

$$HLIN = \sum_{L=1}^{3} HLVAL(L)$$

where

3.a. HLVAL(1): Municipal Services Component

HLVAL(1) is simply the Municipal Services Quality Index CTYDST(1) above with the scale factor PUBFAC(1, K) = 0.25, so HLVAL(1) $\le$ 25

3.b. HLVAL(2): Residential Crowding Index

$$\text{HLVAL}(2) = \text{AMAX1}\,(0.,\ \text{AMIN1}\,(25,\ (\text{CROWD} - 100) * 1.25))$$

where

$$\text{CROWD} = \sum_{k=1}^{3} \text{PROP}(I,\ K) * \ \text{CRDFAC}(K)) / \text{CAP}(L,\ \text{LEV})$$

gives the per cent of capacity reached in the residential structure, PROP(I, K) being the number of population units there of classK, CRDFAC(K) being the amount of space used by a population unit of class K, and CAP(L, LEV) being the space capacity for the building of type L on parcel I operated at level LEV

current data:

CRDFAC(K) = (1.0, 1.33, 2.0)

HLVAL(2) is such that

$0 < \text{HLVAL}(2) < 25$ if $\text{CROWD} < 120$

$\text{HLVAL}(2) = 25$ if $\text{CROWD} \geq 120$,

since AMIN1 takes the minimum of 25 and the per cent overutilization, and AMAX2 takes the maximum of that and zero.

3.c. HLVAL(3): Coliform Bacteria Index

$$\text{HLVAL}(3) = \text{AMAX1}\,(0.,\ \text{AMIN1}\,(50,\ (\text{COLF}(I) - 1000) * \ (.0025)))$$

where COLF(I) is the coliform bacteria count (in parts per million gallons of surface water.[4] In the manner of

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[4] This version is from HSDSSW. Manual #4, p. 90 states $\text{HLVAL}(3) = \text{COLF}(I)/4$, with a maximum of 50 if $\text{COLF}(I) \geq 200$.

HLVAL(2), HLVAL(3) is such that it ranges from zero to fifty if COLF(I) $\leq$ 21,000 and equals fifty if COLF(I) exceeds that limit.

3.d. Summary

$$0 \leq \text{HLIN} \leq 100$$

4. Summary of Environmental and Health Indices

$$\text{PLIN} + \text{HD} + \text{HLIN} \leq 1121.375$$

It would be very unlikely that these indices would reach anywhere near such a great value. On the other hand, if arguments consistently exceeded "desirable" levels by about 10%, with tax rates near 100 mils, the value for this sum should be in the area of 100 to 200. Almost all arguments can range in size from zero to two-hundred, and they tend to yield little or no dissatisfaction in their middle range and below (or above, depending on the direction of the effect). The weights in the indices attached to the deviations above the middle range vary from one-half to two or so for those arguments; these give the rough changes in the index sums with a change in those arguments when they create dissatisfaction. There are two exceptions. One concerns the tax and welfare rates, where weights can drop to one-eighth but absolute values can range much higher. The other exception concerns water quality components, which have non-linear effects.

5. TIM: The Time Index for a population unit on parcel "I"

```
TIM = TIME(6, 2) + 5* TIME(6, 1)
        - (.01)* PRKFAC* TIME(5, 2)
```

where TIME(5, 2) is recreation time expenditure,
TIME(6, 1) is travel time to work,
TIME(6, 2) is involuntary (slack) time,

and PRKFAC is a function of the park use index PKAL(I) for parcel I (see subprogram TMALC) —

```
PRKFAC = AMAX1(0., AMIN1(100, 200
                - PKAL(I)))
```

— so that PRKFAC is 100 if PKAL(I) < 100 and there is no overcrowding, or it is equal to the degree of overcrowding, 200-PKAL(I), when PKAL(I) > 100 and there is overcroding. The scale factor .01 converts this percentage factor to a value ≤ 1.0.

TIM takes its least value when recreation time is maximal, travel and involuntary time are minimal, and there is no park overcrowding. This would be achieved by allocating all 100 time budget units to recreation. Subprogram TMALC (see later chapters) would take out time only for travel to work and illness and assign the residual to recreation; involuntary time would be zero. Under these circumstances, recreation time would be 100 less travel and illness, the latter being one-tenth of the Health Index HLIN. Then,

TIM = 6* TIME(6, 1) + .1HLIN - 100

The lowest limit to TIM, then, comes when there is unemployment (no travel) and minimal health index, at which time TIM = -100. With employment (full time only), and assuming that about 20% of leisure time goes to travel to work as the Washington Study suggests, then TIM would be .1HLIN + 20 here, or 20 if HLIN = 0.

Obviously, the upper limit to TIM comes when there is no recreation time and maximal travel and involuntary time. This can be achieved (under the TMALC procedure, but not according to the manuals) by making no time allocations at all. Then involuntary time would be 100 less travel and illness, so that

TIM = 100 - .1HLIN + 4* TIME(6, 1)

Interestingly, this is maximal when HLIN = 0. Under the 20% travel time assumption the value of

TIM would then be 180. The absolute maximum would come when involuntary time was zero, so that travel is 100 less illness. Then

$$TIM = 500 - .5HLIN$$

which has as upper limit the value of 500.

It is important to note that there are no monetary expenditure items in these indices. Moreover, work (full or part time), education and political time do not figure directly into the indices. Their only utility lies in amassing cash (for transfers) and political/coercive power for influencing the levels of arguments in Environmental and Health Indices, influences which are tentative at best and, as it has been pointed out, will be difficult to simulate effectively because of their bargaining aspect. Since there is no money budget constraint and no penalty in the indices for dissaving, there is no mandatory reason for working and undertaking travel other than to provide funds for transfers. Finally, as will become evident in later chapters, there are other activities in the model for which no activity or travel times are computed.

Allocating to Minimize Dissatisfaction

In the gaming version of the River Basin Model's Social Sector there is no requirement that a population unit (decision-maker) make allocations to minimize dissatisfaction in the Quality of Life Index. The very specification of such a "welfare function" implies that attempts at minimization would be rational and desirable behavior, of course, and it is assumed that if the function is retained in simulated decision-making, then optimum-seeking behavior should be postulated and the appropriate rules of thumb derived. An attempt at a statement of such rules is made in a concluding chapter to this critique.

If a population unit were to try to minimize total dissatisfaction (denoted PDST) represented in the Quality of Life Index, it would attempt to undertake "marginal analysis" of the type applied to utility-maximizing models of consumer behavior. Since the Quality of Life Index is basically a linear function (the pollution component is non-linear, and recreation time interacts with park crowding, as the exceptions), the existence of a single budget (re time) suggests that only a few, and possibly only one, argument under the control of the population unit need be allocated at a non-zero level to minimize dissatisfaction. That education (skill) level and voter registration cannot fall below certain limits for a class further undercuts the importance of those activities. This may not be unrealistic, of course, as the Washington study shows very little interest in these activities is exhibited among the adult population.

Normative allocations in the River Basin Model would all be subject to lack of knowledge of outcomes, of course. Marginal changes in dissatisfaction with changes in arguments are simple enough to identify here, but it is not so simple to see how environmental and health factors will change with population unit time (or money) allocations, especially when effects may be lagged several rounds in the Model's operation. It is true, however, that there are explicit functions in the Model which relate the levels of these arguments to levels of parameters and choice variables in other sectors. If these parameters can be affected by population unit allocations, then these technical functions can be used to replace non-time arguments in the Quality of Life Index of the population unit with functions of its own allocations. For example, water quality could be expressed as a function of treatment level, treatment level as a function of tax revenue, and tax revenue as a function of tax rate and tax base (income, for example). Then if each population unit can specify a desired direction of change in the tax rate and is defined weight in influencing change in proportion to its political activity, the change in the tax rate being the aggregate of these weighted directional changes for all population units in the jurisdiction, water quality as a variable is replaced in the dissatisfaction index by these population unit choice variables, so that marginal returns can be evaluated, conditional on the similar activities and incomes (work) of other population units.

In the absence of such cost/time relations in the dissatisfaction (Quality of Life) index, the population unit, without the option of bargaining, would be prone to try to minimize the Time Index, ignoring the others, even though the discussion above shows that the range of values accorded the Time Index is roughly that of each of the others under normal circumstances. It has already been brought out that when the population unit is unsure of the impact political pressure will have, and when meeting a balanced money budget or saving money for transfers is of dubious merit, then there is little value to a second job and to education and political activity. Similarly, boycott behavior could lead to increased travel time and cost (although, with no money constraint the increased money cost itself yields no dissatisfaction) without the promise of significant change in health or environmental variables. Under these conditions a population unit might be expected to:

1) not request secondary work; this helps keep travel time down, and there is no need to avoid dissaving by earning extra income;
2) not request education or political activity, letting education (skill) level and voter registration drop to their minimal levels for the class; this weakens competitive position for jobs, but again the loss of income does not lead to dissatisfaction; it might lead to greater travel time in job assignment (see EMP), but 3) suggests how to offset this;

3) set the dollar value of time parameter very high to reduce travel time at the expense of cost, which, again, causes no dissatisfaction; this could make it hard to find an acceptable job, under the acceptable limits of travel costs to income earned (see EMP), but unemployment is no problem when there is no money budget constraint—indeed, it would eliminate travel time to work;

4) minimize involuntary time; according to manual # 9, p. 90, this would mean minimizing the disparity between allocations and expenditures realized for second job and education, and this would be done by steps 1) and 2) above, making involuntary time equal to zero; under the TMALC computation of involuntary time as residual time not expended, involuntary time would be minimized by allocating all 100 units to activities without making allowance for the non-choice items of travel and illness; then, even if all demand allocations are not met in the operation of the system, preemption of time for travel and

> illness reduces the chances that there will be slack time; if steps 1) and 2) are followed and only recreation is allocated time (of 100 units), then involuntary time is assured to be zero.

As was suggested in the previous section, then, optimizing the Quality of Life Index by minimizing TIM seems to lead to the allocation of all leisure time to recreation. The fact that this would seem to be so universally the best strategy undercuts the usefulness of choice behavior and the time budget. The most straightforward improvement would be either to impose a strict money budget or attribute dissatisfaction to the level of dissavings, or both. Since recreation time entails consumption expenditures (see OPCM), allocation to recreation cannot be made with abandon under a money budget but requires attention to the job situation, and the same would be true with dissatisfaction to dissavings. Education to improve competitiveness and income becomes a consideration again. In addition, it might be wise to set a limit on the choice of the dollar value of time as some function of the relative times allocated to recreation and to the second job.

Theory suggests that these should be consistent, subject to wage and cost rate parameters, since the dollar value of time is precisely a measure of the marginal rate of substitution between time and money. Indeed, in simulation the dollar value of time to be used in transportation assignments might be a derived rather than a choice variable, being just such a function of relative time allocations.

It seems that the Time Index itself should be expanded to allow education and political activity to be seen as having consumption value in their own right. It has already been suggested that an attempt be made to relate environmental and health arguments to population unit arguments such as income, political time, and so forth, although it is admitted that this may be difficult to do. Finally, there are other activity times and expenses that need to be introduced to the time and money budgets and, possibly, to the Quality of Life. These will be discussed later, and include travel time and cost for shopping and recreation and shopping time itself. As to the nature of the Quality of Life Index itself, its linearity increases the possibility of getting all-or-nothing type solutions such as the recreation allocation case above. Other forms should be considered. These issues will be raised again.

Migration Procedures in MIGRAT

After sector decisions are made and allocations are input to the system and the infrastructure/land-use pattern updated, but before actual assignment procedures are undertaken to translate allocations into actual flows, exchanges or expenditures, the demographic structure of the Social Sector is modified through population unit migration via the MIGRAT subprograms. This means that population unit composition and situation can change between the making of allocations and the determination of expenditures of time, giving dynamism and uncertainty to choice-making. The Social Sector is in a position of continual lag. Migration can be initiated by the level of dissatisfaction of a population unit for the previous round, and it is MIGRAT that calls HSDSSW for computing that level. This means that although the population unit knows the outcomes (expenditures) of the previous round before making current allocations, it does not know the associated level of dissatisfaction, and thus it does not know whether or not the last round's allocations-expenditures led to an improvement or deterioration of its dissatisfaction position before it must make new allocations. This seems to be unnecessary; HSDSSW can be called as part of WRYOU (Social Sector Output) at the end of the previous round. Indeed, in simulating Social Sector decision-making this should be done, since incremental changes in the dissatisfaction components for specific arguments in the previous round may be needed for computation of current adjustments to allocations.

There are six sources for population change in the model, and MIGRAT calls up subprograms to compute the extent of such change and to redistribute the population over the area. Subprogram MOVOUT determines the numbers which vacate housing for reasons of un- or under-employment (these leave the system), dissatisfaction (with information provided by HSDSSW and GETCUT), or random movement. Next, subprogram UNCRWD determines the number who move for reasons of residential overcrowding, usually associated with demolitions. Finally, INMIG computes the number of in-migrants to the system and the extent of natural increase. Subprograms SETUP (using PICKRS) and MOVIN do the actual matching of movers with residential opportunities and the consequent redistribution of the population, and JANCUW and MIGSUM provide printed details and summaries of the migration operations. These procedures will be reviewed briefly with an eye toward their implications for assignments and dissatisfaction levels for movers.

Table 2 summarizes the selection procedure for movers from among the existing population. The estimate of a mover incidence of 20 to 25% is only a rough guess. Most of these remain in the system--about 3% of the original population leave the system because of employment problems. Note that since unemployment or underemployment does not necessarily mean more dissatisfaction because of the way the Quality of Life Index is defined, there may be little overlap of population units falling into those categories of movers. (Underemployed workers are those who must take jobs open to lower status groups; see EMP.) INMIG identifies natural increase as 1.5% of the population and inmigration as 1% of the population plus one population unit for every job which went unfilled the previous

TABLE 2: Determination of Population Units Changing Residences (MIGRAT)

1. Total population at beginning of round (100%)
    - 1.a. 20% most dissatisfied in each class: random one-half move
    - 1.b. 80% least dissatisfied in each class: random % (1, 5, 7) move in each class
2. Remaining approximately 85-90% of population
    - 2.a. under and unemployed: random % (15, 25, 33) move in each class (leave system)
    - 2.b. fully employed: no movers
3. Remaining approximately 80-85% of population
    - 3.a. living on excessively crowded parcels (over 120% crowded): enough move to bring crowding down to 120%
    - 3.b. living on other parcels: no movers
4. Resulting 75-80% of original population retains same residential location

round. This should roughly offset outmigration from the system for job reasons and perhaps provide for a net increase. These population units will also be seeking housing, along with movers within the system. Note that since there are maximal allowable limits to the ratio of journey-to-work costs to income earned, there can be unemployment and unfulfilled jobs simultaneously (see EMP).

Movers staying within the system are selected for consideration in random order by SETUP, which calls PICKRS to choose the new residential location for the population unit. In order to locate on a parcel, the housing quality there must lie within a specified range for the class of the population unit:

Class	Housing Quality Range
low (1)	20 to 70
middle (2)	40 to 100
high (3)	71 to 100

Note that these ranges preclude low and high population units from moving into the same parcel simultaneously. Movers will be located into parcels as long as the percent overcrowding does not exceed 120. The actual selection of a location as "best" for a population unit from among those which are acceptable under these conditions is that location having the lowest Environmental Index. If the reason for the move had been random factors or dissatisfaction, there is the additional condition that this index value must be less than the index value at the previous residence, even if that value was not a crucial factor in the total dissatisfaction; if that condition cannot be met, the population unit must outmigrate from the system.

We see, then, that the specific factors leading to dissatisfaction or employment problems are not considered in choosing a new location. Moreover, a local mover keeps his previous place of employment, even though transportation time and cost — which could have led to the move in the first place — are likely to be much greater in the new location. Reassignment to a new job is possible in EMP, but the procedure biases assignment toward current job, and no distinction is made there between recent movers and non-movers. Moreover, on moving into a parcel the population unit adopts the time allocations of population units of its class already on the parcel,[3] even though these may be quite inconsistent with the work/travel pattern that population unit retains. Under these circumstances, it seems a matter of pure chance that the population unit will have improved its situation. Indeed, it would seem that a population unit could easily enter a continuous cycle of movement, ending only in its exit from the system.[4] The fact that 20% or so of the population units may change residence each round may reflect American life—a move every five years, on the average —but it may also make the model quite unstable.

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[3] In addition, the educational level and voter registration for the in-migration are averaged with those for current residents in MOVIN. Financial information is apparently not transferred unless there are no previous residents. That is an unnecessary omission.

The basic problem with the migration procedures, then, is that they do not systematically match the reason for moving with an appropriate rule of thumb for locating. This precludes any intential tradeoffs by the moving population unit. The utility maximizing models of residential location have been used to demonstrate how activity pattern (time use) can vary as activity (and goods, including land) costs vary by location. Systematic consideration of such tradeoff possibilities allows the population unit to improve its situation in the move. Granted, all moves need not be rational, and the dynamism created by unpredictable change has its realistic features. Yet it does not seem reasonable to divorce completely migration behavior from other population unit choice-making.

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[4] The dollar value of time paramter is important in job assignment, and changes in its value could accelerate or dampen such a cycle.

Here are some suggestions for matching location rules with migration reasons. First, it must be decided whether population units are to move for reasons of "push" alone, or "push" and "pull." At present there is mainly the "push" factor: the decision to migrate depends only on the immediate circumstances of the population unit at its residence with no regard to circumstances elsewhere. An optimizing approach would also consider the "pull" of better circumstances at other locations. The present method considers "pull" only to the extent that location in the system will not occur for certain movers unless some conditions are met; however, if they are not met (i.e., not enough "pull"), the movers still move, but out of the system (i.e., "push" is still the sufficient condition for the move). It is suggested, then, that potential movers be identified first by strength of "push" factors and subsequently by strength of "pull" to other locations. Only those with the greatest "push" should leave the system if no within-system location with sufficient "pull" is found; others should simply stay put in that case. For practical reasons low "push" population units should not be allowed to consider moving even though "pull" may be sufficient at some other location. In theory, a population unit would move whenever it could improve its situation regardless of its current level of well-being, but this is not likely to be feasible in an operational model.

The second issue is the definition of the "push" factors. Theory suggests that the only reason for moving is dissatisfaction. For example, an unemployed population unit that is fairly satisfied should be no more likely to move than an equally satisfied employed population unit. The need for identifying employment reasons in the River Basin Model stems from the omission of a money budget constraint and/or additional arguments to the dissatisfaction index reflecting money expenditures and (dis-) saving and income levels. With those changes, dissatisfaction need be the only criterion to define sufficient "push" levels (by class), with the possibility of retaining some random movement. After all, a utility function should encompass all motivational dimensions for behavior, by definition.

Given sufficient "push" in terms of dissatisfaction, there would be two ways of proceeding. One would be to identify the component index — Pollution, Neighborhood, Health or Time — contributing the most to dissatisfaction (i.e., the one that is the largest) as the predominant "push," or reason for moving. It was suggested that just as movers were differentiated by type of "push," so should the "pull" factor for locating the mover be of the appropriate type. Therefore, the "pull" factor to be considered for locating would be just that same set of variables defining the "push." MIGRAT's use of the Environmental Index is an example of the way the mover dissatisfied with the environment would be located at that parcel at which the strength of the environmental "pull" is the greatest.[5] An example where transportation time in TIM is the push factor would lead to locating where the "pull" of reduced transportation time is greatest. In this instance migration to reduce travel time to work is the complement to job assignment in EMP which attempts to maximize the excess of income over travel costs. The side conditions on crowding and space availability could be retained (the River Basin Model does not allow substitutions involving residential space). The present structure of PICKRS could be easily modified to handle these changes.

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[5] Note that rent level is an argument to the dissatisfaction (neighborhood) index. Therefore, a move to seek a reduction in rent is part of the migration push-pull in terms of dissatisfaction. Space consumption is fixed, however. This is a case where an expenditure item is in the "utility function."

It should be remembered that in theory a mover would evaluate all "pull" factors relative to all "push" factors — that is, it would locate to minimize dissatisfaction. The second approach, then, would be an explicit "marginal analysis" by actually picking that location with space available which minimizes dissatisfaction. Involuntary and recreation time are the only two items in the Quality of Life Index that could not be varied by location to find the optimal one.[6] The data for all other components has already been computed, and it would be hardly more difficult to evaluate total dissatisfaction by location than Environmental Index level, which is currently done. Indeed, all that is involved is the simultaneous consideration of travel time, the Environmental Index, and the Health Index. This seems quite straightforward in a modified PICKRS.

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[6] Other population unit outcome variables which might be added, such as consumption expenditures, might have to be held constant, too. Some, like savings and travel costs, could be allowed to vary, however.

It is felt that these changes are relatively easy to implement without completely altering the structure of MIGRAT. Indeed, in some ways it is simplified, since there would only be dissatisfaction to consider. These changes would also lessen the problem of inconsistencies between work/travel relationships carried over from the old location and the time allocations adopted at the new one, since choice of the new location must take into account the current place of employment. It should be remembered, of course, that times are presumed allocated as optimally as possible by population units, subject to the uncertainties and dynamism of system operation; otherwise, there would be little justification for moving in the first place, other than random factors. Although activity and residential location would still be sequential and not simultaneous as in the neo-classical models, still they would be more closely related and explicitly normative.

Finally, only uncertainty that is felt to be arbitrary has been removed in these modifications. Population units would still make decisions on the basis of previous round costs, prices and opportunities. Just as for non-movers, so also would movers be ignorant of changes in those parameters and the behaviors of other units in the current round, retaining the disequilibrium lag qualities of the model.

SECTION IX
FROM TIME ALLOCATIONS TO TIME EXPENDITURES

There are three types of locational assignments beside migration to be made for Social Sector population units in the River Basin Model. These concern employment, shopping, and park use. Travel times and costs are involved in some of these assignments. In addition to locational determinations there are time determinations to be made for most of these activities, plus time to be spent on education and politics. This chapter will consider all of these except shopping, which will be given detailed treatment in the next chapter on monetary expenditures. In that chapter, modifications will be suggested which will affect the procedures discussed in this chapter.

Full-time employmemt is the first of these assignments to be handled in the model, following migration. EDORD stratifies the population by education level, and EMP and TRTRC make employment assignments based on education (skill) and dollar value of time in travel and update the transportation system. Subprogram EMPRT handles part-time employment assignments and time expenditures, and NSPACK does the adult education time apportionment. Finally, TMALC sets time expenditures for illness, political activity, and recreation. It also determines park use and updates education level and voter registration. These procedures will be discussed in turn.

Employment

The importance of educational level, a proxy for skills, is seen by the fact that EMP takes population units in order of education, highest first as determined by EDORD, in making job assignments. This means that more skilled population units will have first shot at the best jobs. Of course, "best" is relative to the location of the population unit, which must bear travel time and money costs to get to work, so that the best job for a population unit need not be the one paying the highest gross wage. Education level is a function for past expenditures of time in education, so that there is a return to the investment in education in terms of added income due to improved competitive position (place in the education list).

The River Basin Model is socially static, however. First, no population unit can move out of the educational level range of its class, either upwards through additional education, or downwards by foregoing education. These ranges are mutually exclusive by class. Second, jobs are specific to population unit class such that no population unit can take a job open to a higher class; however, "underemployment" can occur when a population unit cannot find an acceptable job in its class and preempts one of a lower type. Salaries are commensurate with job class. These effects considerably dampen the return on the investment in education. One wonders whether building such restrictions on population unit class into the model is desirable. Beginning with a particular distribution of income over the population, the model should be capable of producing a radically different distribution over time due to social mobility. If there is to be class bias in preference for education, this should be accomplished by entering education time in the dissatisfaction index with a weight which is a function of class. Beyond this, educational attainment should only be restricted by financial capabilities to invest. The wealthy would still tend to get more education, but social mobility would not be precluded, and investment in education would be more attractive.

The actual procedures employed by EMP in tracing transportation routes and making assignments and by TRTRC and its entries from EMP in providing trip information and updating the system are too complex to be treated here in detail. In brief, several passes are made through the education-ordered list. First, workers who cannot find a better job within their transportation range are reassigned to their old job, a bias toward keeping that job being built into the procedure. Only then does competition for jobs begin among those workers which might like to change, but a change is made only if a better job still remains untaken by the time the worker is considered on the education list. What will be of major interest here is not this process of competition but rather the way in which jobs are evaluated. It is this evaluation procedure that is tied to time-use preferences and allocations through the dollar value of time in travel parameter, and the travel time derived here is a major component in dissatisfaction and a preemptor of time for leisure purposes.

Full-time employment subprogram EMP is an "optimizer" in that it assigns the population unit's workers to the best job it can, "best" being defined as that job which yields maximal net income after deducting travel costs and a valuation for travel time, subject to a constraint on the travel cost/income ratio. Full-time job shift length is fixed at 80 units for all full-time jobs, wherever the location and whatever the employer, so that work time per se cannot be traded off against other variables. However, any job location might be reached by alternative transportation route and mode combinations with different travel times and costs. Therefore, time and cost can be traded against each other not only by varying the destination of the trip (and thus the gross income earned) but also by alternating transportation route and mode combinations. All these factors must be taken into account in the job selection, and it is the dollar value of time parameter which influences the direction of time-money tradeoffs in this selection. Again, in theory this parameter should also reflect the relative scarcities of time and money in general; that is, it would be inconsistent to save travel time by bearing higher costs when all leisure time is not allocated to activities, nor would it be consistent to reduce costs by taking slower routes or modes if there was no binding constraint on spending. With no money budget constraint, rational choice would always require setting the parameter at an infinite level to obtain as fast a mode as possible. Therefore, the money budget should be added to make the employment assignment procedure consistent with other population unit behavior.

For the population unit of class K residing on parcel I, then, the full-time job selection problem can be viewed as the choice of a job type K^o from among those types $\hat{K} \leq K$ at location J^o among all job locations J, and a trip type L^o among those types L between I and J^o, such that (K^o, J^o, L^o) maximizes

$$SALFAC(I, K, J)* SAL(\hat{K}, J) - CK(I, J, L)$$

subject to

$$AC(I, J^o, L^o) - MXCOST(K)* SAL(\hat{K}^o, J^o) \leq 0$$

where $SAL(\hat{K}, J)$ is the salary offered by the employer at J to workers of population class $\hat{K}$ or above, where $K \geq \hat{K}$; $\hat{K}^o$ is the class of job ultimately chosen, so that if $\hat{K}^o < K$ the population unit is underemployed

SALFAC(I, $\hat{K}$, J) is 1.1 if the population unit worked at J in job $\hat{K}$ last round, and 1.0 otherwise; this scale factor biases choice toward the previous job held

CK(I, J, L) = AC(I, J, L) + VALTIM (I)*AT(I, J, L) is the shadow transportation cost between I and J via trip type L, AC being the dollar cost of the trip and AT its time, and VALTIM being the dollar value the population unit attaches to time in travel

MXCOST(K) is the greatest proportion of income which any population unit of class K is deemed willing to pay for transportation

The expression maximized, then, is the net shadow income or salary, which is the actual salary paid less actual transportation costs and the shadow costs of travel time.[1] Since salaries increase with job type, $\hat{K}^o$ will be the highest paying job type with unfilled positions at employer location J^o, but it need not equal K nor be the highest type for which jobs are available somewhere. Trip type L^o will minimize total shadow trip costs CK(I, J, L) between I and J, but these need not be the lowest in the system. Likewise, of course, SAL(I, $\hat{K}^o$, J^o) need not be the greatest available; it is the net shadow income which is greatest at J^o because of the salary/travel balance there, relative to I. Finally, note that there is a constraint to the problem, whereby at the optimum job location the real transportation costs cannot exceed a certain proportion of the salary earned. This is a somewhat arbitrary constraint, and it is necessary only in the absence of an explicit money budget constraint on consumer behavior. Its purpose is to indicate that workers balk at excessive travel costs and would even refuse to work if no job could offer high enough a salary relative to travel costs. Again, it would be preferable for such behavioral tendencies to result directly from dissatisfaction minimizing behavior without the insertion of rigid conditions like this one. With a money budget, expenditure items in the dissatisfaction index, and a dollar value of time parameter related to leisure time allocations this could be achieved.

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[1] At some places in the manuals, it is implied that the shadow costs of travel time are not included. EMP appears to include them, however.

There are simple ways to handle unemployment. For example, a dummy job location could be introduced with little or no travel time or costs and a salary equal to the unemployment compensation in the jurisdiction (or the travel could be that required to visit a governmental office). Then the choice to become unemployed is an explicit job choice related to other time allocations through the dollar value of time, rather than being a rigid determination. Note, however, that as presently defined the switch from employment to unemployment would create a direct reduction in dissatisfaction of an amount equal to a weighted sum of the welfare compensation and the local tax rates. See discussion of the Neighrobhood Index, above. It is expected that this effect would have to be modified.[2]

Shadow costs of travel time are not actually paid, of course, only real costs are. For any location J, selection of trip type L^o for that location is such that (ignoring the MXCOST constraint)

$$CK(I, J, L^o) - CK(I, J, L) \leq 0$$

for any other trip type $L \neq L^o$. This means that

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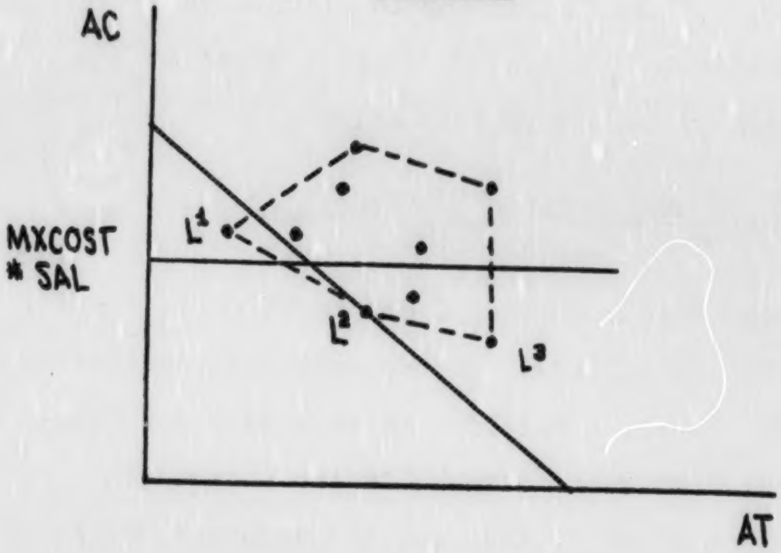
[2] However, the effect on involuntary time could create an increment to dissatisfaction. See the discussion below.

$$-\frac{\Delta AC}{\Delta AT}\begin{cases} \le \text{VALTIM if } L^{o} \text{ is faster than } L(\Delta AT > 0) \\ \ge \text{VALTIM if } L^{o} \text{ is slower than } L(\Delta AT < 0) \end{cases}$$

where ΔAC and ΔAT are the changes in travel costs and time in switching from L^{o} to $L \neq L^{o}$, i.e., those for L less those for L^{o}. Since VALTIM ≥ 0, L^{o} can be slower than L only if its is much cheaper ($\Delta AC > 0$), just how much depending on VALTIM, in relative terms. If L^{o} is faster than L and also cheaper, then the value of VALTIM had no effect in the selection of L^{o} over L, but if L^{o} is faster but more expensive ($\Delta AC < 0$), then VALTIM indicates just how much faster (in relative terms) it must be.

The relationships above apply only if $AC \leq \text{MXCOST*SAL}$ for both L^{o} and $L \neq L^{o}$. The trip selection problem can be illustrated graphically by letting the axes represent travel cost and time and then locating the trip types as points in the interior. An iso-shadow cost contour would be a line with slope -VALTIM; to minimize that cost is to select the trip type for which the intersecting contour is the closest to the origin. However, the constrained optimal trip type must also be located within the cost constraint, represented by a horizontal line at AC = MXCOST*SAL. The optimal trip will be a boundary point (those points connected by dotted lines below). Thus in the figure below, L^1 would be

Figure 1



the unconstrained optimum, L^2 is the constrained optimum, and relations between L^2 and some other feasible point L^3 would be as described above. A lower (absolute) value to VALTIM would mean a flatter cost curve and possible switch to L^3 over L^2. From round to round in the model times and costs change, and a change in VALTIM might not produce the expected change in trip selection. These effects refer to a given location J; changes in these parameters can cause assignment to a new job location as well as a change in trip type and net income.

Assuming that the optimal type of trip L would be chosen between I and J, for any location J, and that the best available job type K would be taken at any given J, then at the optimal location J^o

$$\Delta SAL \leq \frac{\Delta CK - SAL(\hat{K}^o, J^o)\Delta SALFAC}{SALFAC(\hat{K}, J)}$$

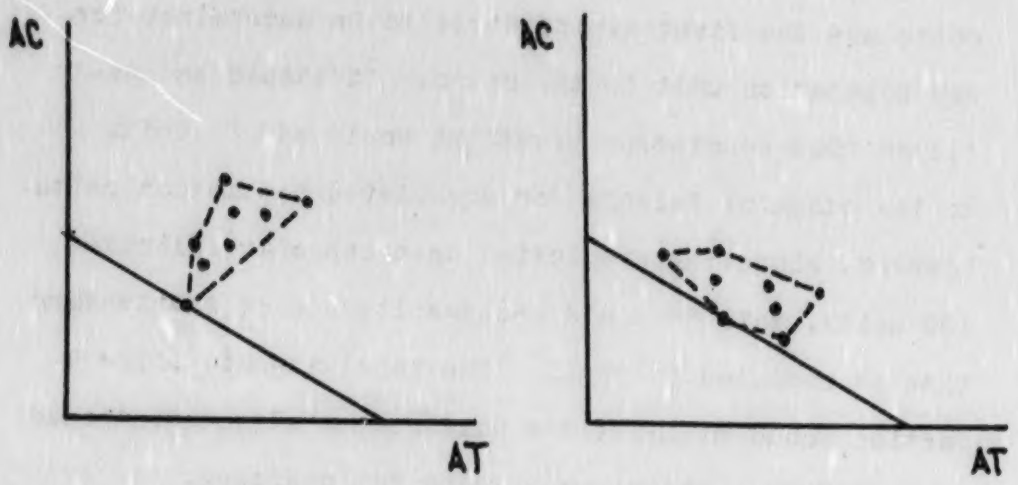
in comparison with any other (job) location J, $J \neq J^o$, where $\Delta SALFAC - 0.1$ if J^o is the same job location as the previous round, + 0.1 if J is that same job location, and 0.0 otherwise. The Δ's are changes in switching from J^o to J, that is, the argument for J less that for J^o. This means, of course, that the change in salary in moving from the optimal location is less than the change in total shadow travel costs adjusted for changes in the salary factor. If J^o is the old job location a greater salary difference is required before a switch in locations is made (i.e., this inequality ceases to hold) than if neither is the old job location. Conversely, if J is the old job location, then an even smaller salary difference is required.

The weight SALFAC was introduced to the assignment procedure to dampen the tendency to change jobs every time salaries changed or travel times and costs changed due to changes in congestion levels at various places in the transportation network. Actual salary received ignores this scale factor, of course. However, a real-world interpretation that could be made would be to consider this as representing the costs of changing jobs, such as lost time at work, expenses in making new arrangements, etc. Under this interpretation it would be appropriate to set SALFAC at unity for the old job and less than unity for a new job. Further, a real charge might be made against the salary received to extract those costs. (Similar costs might also be charged against rent differentials in migration.) The operation of EMP would remain the same, of course.

In its structure the subprogram EMP appears to be entirely consistent with dissatisfaction-motivated behavior for population units, provided that the valuation of time in travel is a true representation of the relative values (in terms of dissatisfaction) of time and money in other activities as well for the population unit, and subject to revision of the MXCOST constraint. The sensitivity of job assignment to VALTIM depends on the nature of the transportation and salary alternatives, of course. Consider a set of jobs offering about the same salary. If the time-cost ratios for the "best" trip type to each are fairly constant, then the jobs will cluster along a line with positive slope in travel time and cost space (Figure 2.a.). Changes in the slope of the shadow cost contour (by changing VALTIM) would have to be drastic to cause a switch in jobs. On the other hand, if the travel time-cost ratios vary considerably (Figure 2.b.), then a slight change in VALTIM could bring about a change in jobs. Such factors as congestion can quickly alter these ratios. With different salaries offered, the cases become more complicated, of course.

In passing it should be pointed out that the impacts of job assignment on the transportation system are lagged. That is, traffic flows are not altered during the job assignment process, so that travel times and costs are fixed for its duration. After assignment,

Figures 2



a. similar time-cost ratios

b. disparate time-cost ratios

however, TRTRC computes new flow levels and congestion factors through CONGES, which it calls. An iterative procedure is used to re-evaluate journey-to-work trip patterns, reassigning population units to revised least shadow cost trip types if necessary in the fact of congestion, but making no changes in the job assignments. Actual travel times and costs assessed the population units are those which result from these iterations, and they could differ substantially from those on which the job assignments were based. Since work time itself is fixed at 80 units, these travel times and costs are the first expenditures to be determined for the population unit in the round. It should be mentioned that consistent treatment would add 80 units to the stock of leisure for unemployed population units. However, since the population unit can only allocate 100 units, this 80 would necessarily add to involuntary time as computed in TMALC. The resulting dissatisfaction would mitigate the welfare/tax effect mentioned above. These matters need to be rationalized.

As it has been pointed out, the population unit factors most directly affecting its full-time job assignment are the dollar value of time in travel and competitiveness in terms of skill (education) level, and only the former can be set independently the same round. Estimating returns to education will be considered later in connection with subprogram TMALC. Education's effect, of course, is to expand the number of job opportunities open to the population unit by giving the population unit a higher priority in the assignment list. Before further discussion of the dollar value of time in travel parameter is undertaken, we will examine the assignment procedure for part-time employment (moonlighting). As a completely discretionary activity, its time allocation should be directly related to relative time/money values and thus to the dollar value of time. Therefore, it will be important that these arguments be consistent here and also in relation to travel behavior for full-time employment. Before part-time job assignment is begun, auto and income taxes and welfare payments are computed through EMPTAX. These will be updated after part-time work determinations are made.

Part-time employment is an activity for which population units make an explicit time allocational request, and assignment subprogram EMPRT will not assign more time to this activity than the population unit requests. EMPRT is similar to EMP in many ways, as it assigns population units to jobs on the basis of maximal net (shadow) income after transportation,[3] where transportation routes (but not modes, as auto trips are mandatory here) are selected on the basis of least cost when time valuation is included. Again, transportation costs are not allowed to exceed a certain percentage of the salary obtained. Here there is no reassignment of travel routes due to congestion.

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[3] Again, the manuals and the source listing seem to disagree on whether or not the income to be maximized deducts the shadow cost of travel time.

There are several significant differences between EMPRT and EMP, however. First, there is the limit that time expenditures cannot exceed the allocation requested. Second, and more fundamental, there is the method of job and time assignment itself. Here, requests for part -time work are considered in increments of ten time units (or the remainder of the allocation, if less than ten). As in EMP, best educated population units are handled first, and the increments of time for each population unit are assigned separately to jobs, each increment being treated as if it were a separate job request. A population unit, then, may have more than one part-time employer and make several part-time job work trips. If assigned in succession, increments for a population unit would go to the same job as long as the demand for labor there were unmet. However, the order of assignments here is to attempt to assign single increments for all population units in the list requesting them before assigning additional increments for any one population unit, running through the education-ordered list as many times as is necessary until no more assignments can be made. This procedure greatly increases the chances that different increments for a population unit will go to different jobs. This procedure has the effect of reducing the differences in competitiveness for jobs which would be in effect if request were considered in their entirety instead of incrementally. The result will be less disparity in incomes earned and travel costs borne. This may be admirable in a normative sense, but it does not seem very realistic.

In each job assignment for the population unit the role of the dollar value of time is the same as in EMP, permitting the population unit to bias assignments one way or the other in terms of relative travel time and cost. Separate trip times and costs are summed to give totals for part-time work. As with full-time work, no allocations are made for travel time. This time may be allowed for by not allocating all 100 time budget units, but this is not necessary since no disutility arises from allocation-expenditure disparities as long as all time is expended.[4] There is the equivalence of 80 units of part-time work and one full-time job (which takes 80 time units). Consequently, network travel times and costs for full-time jobs are scaled by one-tenth for each ten unit time increment assigned to a part-time job. Comparable adjustment is made for salaries, which are then summed to give part-time income.[5] The implication, then, is that there is a given number of round trips to be made per unit of time worked, in full- or part-time jobs, each trip having the same time jobs, each trip having the same time and cost (specific to route and mode, of course).[6] Note, however, that part-time assignments are made after the TRTRC (CONGES) system updating for congestion, so that actual times and costs by link may be different from those in full-time assignment.

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[4] Again, the manuals and source listings disagree on this point. In any case, since pre-emptions for travel tend to come largely from recreation where there is no disparity penalty, the net effect may be the same (see TMALC).

[5] Although salaries bear this proportionality relationship, employers can make independent requests for full-time and part-time labor.

[6] These implicit frequency relationships are of importance in considering relative times and costs of travel in other activities.

If there are not enough acceptable jobs (in terms of the travel cost/salary rule, which perhaps should be dropped with the addition of a money budget constraint, etc.), some of the population unit's allocation of time to part-time work will go unexpended. As with all activity time allocations, such a residual increases the chance that there will be slack time remaining and dissatisfaction due to involuntary time. Part-time jobs are strictly limited to the appropriate population unit class; no underemployment is allowed. Competitiveness (education level) is thus especially important here, for dissatisfaction can be direct as well as indirect (through the loss of income; there is no involuntary time component associated with full-time work at present). EMPTAX is called to compute appropriate increments to the income and automobile taxes, as in EMP. EMPRT returns TIME(1, 2) as the amount of part-time work obtained. The total journey-to-work travel time from EMP and EMPRT is TIME(6, 1). These times will be used in further computations in TMALC. Money items (incomes, travel costs, and taxes) will be reviewed in the next chapter on the money budget in connection with subprogram WRYOU.

Consistency between job assignment and leisure time allocation hinges on the determination of the dollar value of the time spent in travel. In this discussion of the model, we have not been careful to distinguish between a general dollar value of time in leisure and the dollar value of time in travel, but in theory they can be different. The best way to illustrate this and to show the relation between optimal work/travel conditions and leisure time allocations is to set up a simple simulataneous solution, utility maximizing problem similar to the ones presented in the first part of this report but oriented toward the type assignments or choices considered above.[7] This will be a "fictitious" problem, in that location and route/mode variables are treated as continuous variables evaluated with perfect knowledge. The appropriate problem is to

$$\text{maximize} \quad U = \mathcal{U}(x, t_1, t_r)$$

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[7] A more complete example for the River Basin Model is set out in a later chapter, with explicit leisure activities and activity costs.

subject to

$$80w_1 + t_2w_2 - 80\beta_1 - t_2\beta_2 - x \geq 0 \qquad \text{(money budget)}$$

$$100 - 80\alpha_1 - t_2\alpha_2 - t_2 - t_1 \geq 0 \qquad \text{(time budget)}$$

$$t_r - 80\alpha_1 - t_2\alpha_2 \geq 0 \qquad \text{(travel time)}$$

where x is consumption expenditure (income after transportation when the money budget constraint is binding),

t_1 is time in (non-travel) leisure,

t_r is total travel time (journey-to-work, full and part-time)

w_i, α_i and β_i are wages, travel times and travel costs per unit of time spent in work of time i, i = 1 for full time and i = 2 for part-time,

t_2 is the amount of part-time work time chosen.

As in the River Basin Model there is no (dis-) utility to work per se (which is 80 units for full time jobs) but there is for travel to work. First order conditions for x, t_1 and t_r are

$$\frac{\partial u}{\partial x} - \lambda \leq 0 \qquad \begin{cases} = 0 \text{ if } x \geq 0 \\ < 0 \text{ if } x = 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_1} - \theta \leq 0 \qquad \begin{cases} = 0 \text{ if } t_1 \geq 0 \\ < 0 \text{ if } t_1 = 0 \end{cases}$$

$$\lambda(w_2 - \beta_2) - (1 + \alpha_2)\theta - \alpha_2 \psi \leq 0 \quad \begin{cases} = 0 \text{ if } t_2 \geq 0 \\ < 0 \text{ if } t_2 = 0 \end{cases}$$

where λ is the shadow price on money (income), θ is the shadow price on non-work leisure time (stock of 100 units), and ψ is the shadow price on travel time. These three conditions would hold for optimal allocation of x, t_1 and t_2 for any given choice of locations for full and part-time work. The third condition can be written as

$$\frac{\psi}{\lambda} \leq \frac{1}{\alpha_2} \left\{ (w_2 - \beta_2) - (1 + \alpha_2)\left(\frac{\theta}{\lambda}\right) \right\}$$

which means that at the optimum only if the dollar value of time in leisure

$$\frac{\theta}{\lambda} < \frac{w_2 - \beta_2}{1 + \alpha_2}$$

will there be a positive amount of time allocated to part-time work when there is disutility to travel (see below). The term on the right is a net wage rate (after travel) per unit of time (in work and travel) for part-time work. Remember that (θ/λ) is (minus) the marginal rate of substitution of leisure for consumption expenditures:

$$\frac{\theta}{\lambda} = \frac{\partial \mathcal{U}/\partial t_1}{\partial \mathcal{U}/\partial x}$$

Finally, if there is part-time work, then the dollar value of time in travel (ψ/λ) is proportional to the difference between that net wage for the part-time job and the dollar value of time in leisure:

$$\frac{\psi}{\lambda} = \left(\frac{1 + \alpha_2}{\alpha_2}\right)\left\{\left(\frac{w_2 - s_2}{1 + \alpha_2}\right) - \left(\frac{\theta}{\lambda}\right)\right\}$$

ψ itself turns out to be the negative of the marginal utility of travel, since the first order condition for t_r is

$$\frac{\partial \mathcal{U}}{\partial t_r} + \psi \leq 0 \qquad \begin{cases} = 0 \text{ if } t_r \geq 0 \\ < 0 \text{ if } t_r = 0 \end{cases}$$

where, if ψ is to be positive, there must be dis-utility to travel. In the River Basin Model there is just such dis-utility (dissatisfaction), although the linear function and the absence of utility for consumption expenditures and of the money budget constraint in that model undermines the consistency desired here, as it has been pointed out several times. The dollar value of time in travel, then, is just (minus) the marginal rate of substitution for travel for money. If travel yields disutility at the optimum, then this is in effect complementarity rather than substitution.

$$-\frac{\partial \mathcal{U}/\partial t_r}{\partial \mathcal{U}/\partial x} = \frac{\psi}{\lambda}$$

These derivations give the origin of the dollar value of time in travel and its relation to leisure versus income tradeoffs. It remains to be shown how that parameter figures into job selection and choice of trip type. This can be done here only in a suggestive fashion. Let l_i be a continuous "location" variable for work of type i (i = 1 for full-time work and i = 2 for part-time), and let m_i be a similar variable for choice of trip type (mode/route combination). Wage w_i is a function of location variable l_i, and travel times and costs α_i and β_i per unit time in work are functions of both l_i and m_i. Then there are the additional first-order conditions for i = 1, 2

$$\lambda \left\{ \frac{\partial w_i}{\partial l_i} - \frac{\partial \beta_i}{\partial l_i} \right\} - (\theta + \psi) \frac{\partial \alpha_i}{\partial l_i} = 0$$

$$-\lambda \left(\frac{\partial \beta_i}{\partial m_i} \right) - (\theta + \psi) \frac{\partial \alpha_i}{\partial m_i} = 0$$

The first says (after dividing through by λ) that at the optimal job (location) the change in the net wage rate after travel costs per unit work time are deducted should equal the change in travel time valued at the sum of the dollar values of time in leisure and in travel. Similarly, the second condition says that at the optimum a change in the travel cost with a marginal change in the trip type (as a fictional continuous variable) should equal the negative of the marginal change in travel time valued associated at the sum of the two dollar evaluations of time.[8] When $t_2 > 0$ one of these evaluations may be expressed in terms of the other, as above. Finally, the two conditions for work type i can be combined to show that at the optimum the ratios of dollar changes and time changes for location (job) and trip type should be equal:

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[8] Presumably the partials of travel cost and time with trip type are of opposite sign at the optimum; one cannot be reduced there without increasing the other.

$$\frac{\dfrac{\partial w_i}{\partial l_i} - \dfrac{\partial \beta_i}{\partial l_i}}{\dfrac{\partial \beta_i}{\partial m_i}} = - \frac{\dfrac{\partial \alpha_i}{\partial l_i}}{\dfrac{\partial \alpha_i}{\partial m_i}}$$

Obviously, the assignment procedures in EMP and EMPRT produce results which can at best only approximate a perfect knowledge, continuous variable solution such as this. The point of this discussion, however, is that the best strategy for the population unit is to use its best estimates of travel times and costs and wage rates -- as perhaps those of the previous round modified for trends or anticipated changes -- in jointly determining the times to be allocated to part-time work and other leisure activities and the value to be given to VALTIM to produce consistent trading off of travel time and costs and job salaries. From first-order conditions such as those above (or the more detailed ones in a chapter to follow) rules of thumb for simulation of this decision-making by population units can be developed. In passing, we have discovered that a proper interpretation of VALTIM attributes to it both a valuation of time in leisure as an opportunity cost and a valuation of time in travel per se as a dollar evaluation of the dissatisfaction arising there.[9]

Adult Education

The subprogram which processes population unit requests for time in public adult education is NSPACK (night school). It does not make final time assignments (those are done in TMALC) as EMPRT does; instead,

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[9] Compare

$$\frac{\partial \beta_i/\partial m_i}{\partial \alpha_i/\partial m_i} = -\left(\frac{\theta + \psi}{\lambda}\right) \text{ with } -\frac{\Delta AC}{\Delta AT} \simeq \text{VALTIM}$$

for trip choice, and for job choice, compare

$$\frac{\partial w_i}{\partial l_i} - \frac{\partial \beta_i}{\partial l_i} = \left(\frac{\theta + \psi}{\lambda}\right)\frac{\partial \alpha_i}{\partial l_i} \quad \text{with}$$

$\Delta SAL - \Delta AC \simeq VALTIM * \Delta AT$ (assuming SALFAC = 1 and $\Delta SALFAC = 0$).

it modifies time request on the basis of night school capacity in the population unit's political jurisdiction. The ultimate time expenditure comes from a further modification in TMALC on the basis of time availability in the population unit's time budget. There is no spatial assignment per se, as there is assumed to be only one facility per jurisdiction and that is the sole opportunity for the population of the jurisdiction. Locational factors come up only in location of the facility (political pressure by users) and migration (population unit location relative to opportunities as "pull" factors). Private education is assumed to be available in sufficient capacity so that no modification in requests is warranted prior to TMALC's treatment.

It is assumed that the highest class population units will not allocate time to public adult education and the lowest will not allocate time to private adult education (the former type is free and the latter available only at a cost; all population units support public education through taxes). These presumptions are built into NSPACK as rules, and they do have some descriptive validity. Once again, however, it is argued that such effects should be demonstrable outcomes of the choice model -- that is, differences in investment in education by class should come from differing preferences (which could reflect different subjective discount rates) and/or differing financial capability relative to cost-return differences by type of education[10] -- without the necessity of ruling out any particular type of choice by fiat. One modification which may help in this regard is to assign a greater return to private education; at present both have identical effects on skill level (see TMALC).

As with most types of facilities in the River Basin Model, educational facility capacity is a function of the level of employment, and employment is constrained by the number of jobs offered, in connection with the size of the physical plant and its condition, public finances and policy, etc. Specifically, the capacity for public adult education in a jurisdiction is

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[10] Time budget analysis is directly applicable to analysis of the returns to education. Gary Becker, among others, has made a start in this direction. See his, Human Capital (New York: Columbia University Press), 1964.

$$NCAP = 5*(\sum_{K=2}^{3} K* OBTN(K, 2))$$

where OBTN(K, 2) is the number of part-time employees of population unit class K in the facility for the jurisdiction. The scaling factor for adjusting time allocations of population units in the jurisdiction is

$$RATIO = MIN1(1.0, NCAP/ILD)$$

where ILD is the sum of allocations (requests) TIME(I, 2, 1) in the jurisdiction. Then for a population unit on parcel I in the jurisdiction the tentative time expenditure in public adult education is the scaled allocation

$$TIME(I, 2, 2) = RATIO*TIME(I, 2, 1)$$

This expenditure is subject to further modification for time availability in TMALC. The sum of these assignments

$$\begin{aligned} USE &= \sum_{I} TIME(I, 2, 2) = RATIO*ILD \\ &= MIN1(ILD, NCAP) \end{aligned}$$

is the minimum of the capacity and the summed requests and is stored as the total demand on the facility. (Apparently this figure is not corrected later when final population unit time expenditures are determined in TMALC.) These are very simple supply-demand relationships where excess demand brings about proportional reduction of all demands until gross demand equals supply. Supply (capacity) can only be increased through increased tax revenues and political pressure for altered public policies affecting the employment level.

In the current version of the model there is no charge against time or money stocks for travel for purposes of public education, if undertaken. Since there is but one public education facility location in each jurisdiction, the computation of such charges would be quite simple, and they should be included. As with work, these charges should be determined on a per trip basis and scaled to be proportional to education time received (in terms of the final time expenditure to be derived in TMALC). And as in that case, the scaling factor will include implicitly a parameter giving the length of an educational episode each occurrence (trip) so that frequency is really the underlying choice variable. Mode and/or route selection could be of the type considered for work, reflecting the parameter VALTIM. Unless dissatisfaction of travel is thought to differ by trip purpose, TIME(I, 6, 1) travel time should be incremented to include the education activity trips. Similarly, EMPTAX should be called to compute the auto tax, if any, to be added into the population unit financial statement.

There is no direct satisfaction accruing to time spent in education at the present. Its only benefit is indirect through the increase in the skill level it provides and thus the possible improvement in competitiveness for jobs it yields. This improved competitiveness becomes manifest as increased net income over successive rounds, as the returns to the investment in education. There is a limit to this improvement (see TMALC); moreover, it may take educational expenditures just to hold even, as skills are assumed to decline with time. Again, there is a lower limit to this decline. Since estimating the future income and its present value is not easy to do in this model, further discussion of the feasibility of adult education will be postponed until a full-blown optimization model is presented in a subsequent chapter. It can be expected that for education to be undertaken the marginal returns in (the present value of a future stream of) income to education must be positive and equal the marginal costs incurred, including the opportunity cost of leisure foregone, direct costs in taxes (if variable), and the dissatisfaction of the associated travel. These, of course, involve the shadow prices (and dollar values) of time in leisure and travel, and consistent rules for simulation in time allocation to education can be developed as part of the general population unit choice strategy.

Charges Against the Time Budget

Subprogram TMALC checks part-time work, travel (to work) and public education time assignments against the stock of time available. It also determines expenditures for illness, private education, political activity, and recreation, again with an eye to time availability. The education time expenditures are used to update the skill level of the population unit and the political time expenditures to update voter registration, both done in TMALC. TMALC also determines park use in recreation activity, and recreation time becomes a factor in consumer expenditures (see OPCM). Some of these time arguments, and the involuntary time also computed in TMALC, are those used in HSDSSW in determining the Time Index measuring the dissatisfaction resulting from the outcome of time allocation/expenditure assignments. These manipulations will be discussed in turn below. It should be pointed out here that there are omissions in time expenditures which should be corrected. Such omissions include time for shopping and for travel for shopping, political activity, public and private adult education and recreation. Including these would involve some changes in TMALC, and these will be introduced here and in the next chapter.

The first computation made for the population unit on a parcel by TMALC is that for time spent in illness. This is simply one-tenth of the Health Index, HLIN, for the parcel of the population unit. Under extremely adverse circumstances this could take up a sizeable portion of the leisure time budget (see HSDSSW). It is indicative that illness time is to be charged against the stock of leisure (100 units) only, not against full-time work. Just as work time under unemployment should be credited to leisure time (see EMP), so should illness as a "leisure" activity have an impact on full-time work behavior; otherwise, the individual bears most of the costs of illness and institutions employing them bear little.[11] A simple formula would be to reduce full-time work (and thus the salary earned and the travel time and costs incurred) by four-ninths of the illness time, and to charge the remaining five-ninths against the leisure stock, these being in the proportion of full-time work time to total leisure time. Finally, as will be seen in WRYOU, there is no direct money cost to illness for the population unit. Charges proportional to time could be included easily enough, as the costs of medical attention (cf. HLTCST).

The first time budget accounting made by TMALC is the deduction of illness—

HLTVAL = HLIN/10

---

[11] Illness does cut into time available for part-time work, hence labor supply, and for private education and recreation, hence demand for services. Lost income in part-time work likewise reduces consumption expenditures (consumer demand) when a money budget is included. If work times are modified for illness, establishment employment and capacity will need revision.

—then travel to full- and part-time work (TIME(6, 1)), and time obtained in part-time work (TIME(1, 2)) from the stock of 100 time units.[12] The residual is

```
REMTIM = 100 - HLTVAL
             - TIME(6, 1)
             - TIME(1, 2)
```

Only if this remaining time is positive will there be time available to assign positive time expenditures to any other leisure activities. It does not appear that travel or part-time work are readjusted in the (unlikely) event that this remainder is negative. Theoretically, this should be done by reducing the time in work and travel appropriately for the population unit's part-time job increments, starting with the one assigned to it last, and such reductions should be charged against the empoyer's labor input as well as against the worker's time and net income. Such readjustments may become tedious and be deemed not worth the effort required, however. Again, part-time work obtained in EMPRT for the population unit will not exceed the amount requested:

---

[12] Questions have been raised elsewhere about the appropriateness of a uniform stock of 100 units regardless of employment or labor force participation status.

TIME(1, 2) ≤ TIME(1, 1)

The strict inequality holds if not enough acceptable part-time jobs were available for the population unit.

The next accounting handled by TMALC involves time in public education. NSPACK may have modified the population unit allocation for this activity for capacity limitations, and TMALC checks on leisure time availability, setting

```
PUBED = MINO(REMTIM, TIME(2, 2))
TIME(2, 2) = PUBED
```

so that the new public education time expenditure is the minimum of the remaining stock of leisure time and the capacity-modified request. This is the general method to be used in checking time expenditures against the remaining stock of leisure time.[13] The next time allocation to be considered is that for private adult education, remembering that the model will not allow the highest class groups to choose public adult education. First, the residual leisure stock is decremented for public education as

---

[13] If REMTIM should ever become non-positive, the model will treat all subsequent time expenditures as being of zero level.

```
REMTIM = REMTIM - PUBED
```

and private educational time expenditure is determined as the minimum of this residual and the original allocation:

```
PRVED = MINO(REMTIM, TIME(3, 1))
TIME(3, 2) = PRVED
```

There is no capacity constraint on private education, which is provided at a cost to the population unit (see WRYOU). As with public education, there have been no travel costs or times computed for private adult education. Unlike public education, however, private education seems to be a fictitious industry: there is no facility for it in the model and therefore no travel can be determined. This omission is not crucial, but it does leave a gap in the system. In any case, travel should be computed for public education, as suggested previously. If the time budget limits educational expenditures, then travel for education would have to be scaled down together with public education time. These times in education are used to update the educational (skill) level of the population unit (below).

The last two time activity expenditures to be computed are those for political activity and recreation. Neither expenditure subjects the original allocation to any restriction other than time availability. Thus the remaining stock is calculated and the expenditures set in each case:

```
REMTIM = REMTIM - PRVED
POL = MINO(REMTIM, TIME(4, 1))
TIME(4, 2) = POL
REMTIM = REMTIM - POL
RECR = MINO(REMTIM, TIME(5, 1))
TIME(5, 2) = RECR
```

The subprogram then computes the final stock residual not expended and stores it as "involuntary time:"

```
REMTIM = REMTIM - RECR
TIME(6, 2) = REMTIM
```

It is this involuntary time which creates direct dissatisfaction in the Time Index. Note that if involuntary time is positive, then time allocational requests were met for at least private adult education, political activity, and recreation; this is,

```
TIME(K, 2) = TIME(K, 1)
```

for K = 3, 4, 5. Because they involve external constraints, time expenditures for part-time work and public education may be less than the times allocated to them even if there is slack leisure time.

The discussion of HSDSSW in the previous chapter explained how involuntary time could be minimized by leaving no time for illness or travel and allocating all time not desired in education, part-time work or politics to recreation. (Some confusions on the definition of involuntary time were also brought up there.) This type of allocation procedure is almost certain to produce a restriction on recreation time, but since that would yield no positive involuntary time and since there is no penalty to allocation-expenditure discrepancy for recreation, the results are desirable. One qualification is that with a money budget constraint introduced to the problem the population unit may not be able to afford so much recreation (see OPCM, next chapter) if it must constrain its allocations in terms of money as well as time. The simplest solution lies in the inclusion of a dissavings component in both the money budget and the dissatisfaction index. Then the money budget will always be binding by definition, and allocations can proceed as at present relative only to a fixed time stock, with the knowledge that there will be a loss of satisfaction if dissaving is positive.[14] With a money budget the time subprogram TMALC and the money bookkeeping subprogram WRYOU would have to be integrated in the event that there were an absolute limit to dissavings. Then time and money assignments would proceed jointly, ceasing if either time were exhausted or the dissavings limit reached. In case of the latter, involuntary time would measure the slack (unexpended) time left. Without a limit to dissavings the present separation of time and money accounting could be maintained.

These issues raise a further question about the time assignment, and that concerns the order in which leisure activities are processed for assignment in TMALC. To force a strict ordering such that excessive transportation times and illness are all incurred at the expense of recreation alone as long as it is non-zero may be quite unrealistic. Indeed, it is likely that demand for recreation becomes exceedingly inelastic while it is still a major component of leisure time, so that reductions in other activities become increasingly probable. Perhaps the issue is one of interpretation. If education, political and part-time work activities are seen to involve commitments which are irrevokable for the course of a round ("the short run"), then the current priorities are appropriate, forcing readjustments to be made in allocations between rounds. From a behavioral point of view, on the other hand, it seems safe to say that few people, individually or in the aggregate, would adhere strictly to commitments which entailed great losses of satisfaction in other areas, even in the short run. The tenuous role of education and political activity (and even of work when there is no money budget or dissatisfaction for dissavings) in yielding satisfaction further suggests that they, not recreation, would be reduced by the population unit. A final issue which is related is that there is something unreal about forcing the population unit to stick to a recreational allocation in the event that it has slack time and money. Involuntary time really makes sense only when a money constraint on expenditures is reached. With extra money or credit that time should go to recreation.

---

[14] In simulation, the rules derived for making those time allocations would of course reflect prices, costs, wages, and dissatisfaction for dissaving. The point is that money allocations would follow automatically just as they do at present.

It is recognized that these procedures in TMALC and other subprograms which have received criticism here were developed with an eye to operationalism as much as to behavioral realism. It is not suggested that changes be made without weighing the costs of such changes against the presumed improved realism to be obtained. Time allocation in a simulation model is novel and provocative, and every effort should be made to isolate behavioral issues in the simulation and to resolve them as realistically as possible for modeling purposes.

Educational and Political Status

In addition to determining time expenditures, TMALC updates the educational and political status of the population unit on the basis of those time expenditures, and it assigns the population unit to parks for recreational activities. The increment to the educational level of a population unit of class K as a function of time in education is simply the product of the total time in adult education this round times a productivity factor for that time. The education level ILEV of the population unit is thus incremented to be

```
ILEV = ILEV + (PUBED + PRVED)/TUE(K)
```

where TUE(K) is the inverse of the productivity factor for class K. Current data are (6, 6, 8), so that the highest class (K = 3) has the lowest productivity factor (1/8). Each class is restricted to a range of educational levels, however, and these ranges do not overlap; hence the lack of social mobility in the model (see the discussions of NSPACK and EMP). Thus ILEV must be limited to the appropriate range:

```
ILEV = MINO(NELEV(K + 1),
       (ILEV - NELEV(K))*9/10 + NELEV(K))
```

Here NELEV(K) is the minimum education level of class K (and NELEV(K + 1) is its maximum level. Current data for NELEV(K) are (0, 40, 20, 100), where K = 4, is a "dummy" class. There is a built-in decrement to ILEV for the passage of time. This is represented by the use of the factor 9/10 rather than unity to scale the difference between the lower limit and the level adjusted for time in education. The result is that the new educational level is the minimum of the upper limit for the class and the lower limit plus 9/10 of the adjusted amount above the lower limit. If no time is allocated to education (PUBED + PRVED = 0) the decrement in ILEV due to the passage of time is

$$\Delta \text{ILEV} = -\frac{1}{10}(\text{ILEV} - \text{NELEV(K)}) \leq 0$$

Since this decrement gets (absolutely) smaller as NELEV(K) is approached, strictly speaking it is possible to approach NELEV(K), but it cannot be reached if at any time ILEV were greater than NELEV(K). In addition, to maintain a constant educational level (ΔILEV = 0), it takes

$$\text{PUBED} + \text{PRVED} = (\text{ILEV} - \text{NELEV(K)})*\text{TUE(K)}/9$$

units of education each round.[15] Again, this approaches zero as ILEV approaches NELEV(K).

The cost of public education to the population unit is not easy to compute, since there is not a one-to-one correspondence between government revenue sources

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[15] This equation produces values roughly one-half those given in the examples on p. 88 in manual # 4. The reason for the discrepancy is not clear.

(taxes) and expenditures. However, a unit of private education time costs 3000, so that to increase the education (skill) level by one unit through private education costs

3000*TUE(K)*(10 + ILEV - NELEV(K))/9

Since class type constricts the educational level of the population unit, the list produced by EDORD is automatically ordered by class as well as by education level within classes. This means that a population unit cannot alter its competitive positive with respect to population units of a higher class. Note that the relative values of TUE(K) imply decreasing returns to scale on the investment in education; however, this is of no practical consequence to population units, since TUE(K) is constant within the educational range for any given population unit. The decreasing returns phenomenon also works in reverse, so that it takes less education to maintain a constant level at higher levels.

Earlier in this chapter an argument was given for removing these class limits on education level. To retain decreasing returns to scale in that event — indeed, to make it a real factor in the time allocation problem - TUE should be made an increasing function of ILEV regardless of class. Again, it should be pointed out that PRVED and PUBED yield the same returns but at widely different costs. This effect may need modification. One approach would be to make the rate of return to public education a decreasing function of the demand/supply ratio but allow demand to exceed supply (capacity), i.e., not scale down the time allocations. Then private education could become economically feasible in comparison with public education when the quality of the latter declines through over-utilization. Another approach would be simply to assign excess demand for public education to private education, so that there are direct as well as indirect costs to the population unit for overutilized facilities. These changes are relatively minor but could be helpful in making education a more dynamic force in the model.

Time expended in political activity increases the percent of the population unit's members who are registered to vote. This effect only lasts one round, however. Moreover, the base of eligible persons is a constant and varies by the class of the population unit. The increment to the registration rate of the population unit is

$$\text{IREG} = \text{PB} + \text{PS}*(\text{POL} - \text{PT})/100$$

where PB is a base rate, PS is a variable rate and PT is a base value relative to the political time expenditure POL. PB and PS are likewise functions of POL. The parameter values are

POL	PB	PS	PT
$0 \leq \; < 10$	0	70	0
$10 \leq \; < 50$	7	7	10
$50 \leq \; < 60$	10	50	50
$60 \leq \; < 100$	15	7	60
100	15	10	100

There are some marked pecularities about IREG. First, since PS/100 gives the slope of the graph of IREG for a range of the values of POL, it is apparent that the ranges of POL between 0 and 10 and between 50 and 60 are of special importance. There the slopes are .7 and .5, respectively, whereas in the other ranges the slope is but a fraction of those values, being .07. Second, since $\text{POL} \leq 100$, IREG for $\text{POL} = 100$ is only 15 (the same as for $\text{POL} = 60$), but between

POL = 60 and POL = 99 IREG rises linearly to IREG = 17.73. Presumably, the relation of the PB values to those for POL (or PT) is meant to indicate decreasing returns to political activity. By breaking IREG into a series of linear segments anomalies have been created. It would have been simpler just to use a single non-linear function over the whole domain of POL.[16]

The updated voter registration is computed as

```
VTRS = ((IREG + 100)* NREG(K))/100
```

where NREG(K) gives the base number of registered voters (a minimum) for class K population units, the data being (100, 140, 200) for K = 1, 2, 3. These figures indicate that the minimal registration in the highest class population unit is twice that of the lowest and over 40% more than that of the middle group. Even with maximal political time the lowest group can increase its registration by only 18 voters. There seems to be an inconsistency when the same maximal effort can yield such disparate results by class; it can hardly be argued that lower class population units should behave against their own desires more than upper class ones. Again, the suggestion being offered here is that differences in voter registration be allowed to arise from differences in time allocations rather than being built into the model. Then, if observable differences in registration rates are seen to stem from differences in the choice-making environment by class, the model will still be able to reproduce them without ruling out other possibilities. Under rational time allocation it can be expected that for political activity to be undertaken the marginal returns however measured must be positive and not be less than the marginal costs involved, if only the dollar value of the leisure (recreation) foregone. Measurement of such returns is difficult, of course, and they could incorporate "preferences" plus subjective probabilities that the activity will lead to desired results. These problems will be considered again in this report.

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[16] This is true in many instances in the River Basin Model and only adds to the complexity of the programming.

Park Use Assignment

The last task of subprogram TMALC is the assignment of population units to parks for their recreational activity. This assignment is of special relevance in time allocation, since the park use indices that result are components in the park factor by which recreation time expenditures for a population unit are weighted in the Time Index. A population unit is thought to make use of only those parks located within a five-by-five parcel grid centered on the residence of the population unit. The effective space available at a park is the "normalized" supply there; e.g., for park J in the grid centering on parcel I, the supply PRKN(J, 1) is

```
PRKN(J, 1) = PARK(J, 1) + 2*PARK(J, 2)
```

where PARK(J, L) gives the amount of undeveloped (L = 1) and developed (L = 2) land in the park. Hence developed land is twice as effective, or has twice the capacity, as undeveloped land. The sum of the normalized supply over all parks J in the grid gives the total park supply available for population units at the central location I:

$$\text{FLND(I)} = \sum_{\substack{\text{J} \\ \text{in grid} \\ \text{on I}}} \text{PRKN(J, 1)}$$

Persons at I make use of park J in the grid on I in proportion to the share of the park supply or capacity available to I that J presents. Therefore, the number of people at I using park J is

$$\text{PEOPLE(I, J)} = 500*(\text{PRKN(J, 1)}/\text{FLND(I)}) *(\sum_{\text{K}} \text{NMPU(I, K)})$$

where NMPU(I, K) is the number of population units (of size 500) of class K living at I. The total demand for (use of) park J is found by summing these assignments to J over all parcels I in the grids of which J lies, i.e., all I in the five-by-five parcel grid centering on J:

$$\text{PRKN(J, 2)} = \sum_{\substack{\text{I} \\ \text{in grid} \\ \text{on J}}} \text{PEOPLE(I, J)}$$

Finally, a park use index is computed for each park J as the ratio of demand for that park through these assignments to the capacity of the park. For park J the park use index PKUSE(J) is

PKUSE(J) = PRKN(J, 2)/(PRKN(J, 1)* .01)

The supply figure in the denomenator is multiplied by .01 to convert it from a percentage, in which it was originally computed, to a proportion.[17]

The park use index used in HSDSSW to scale recreation time for a population unit on parcel I is PKAL(I), the minimum of the park uses indices PKUSE(J) for the parks J in the grid centered on I. There are several shortcomings to the derivation of these indices and their use in this fashion in the dissatisfaction index. Perhaps the most serious is that park use is completely independent of recreational time expenditures, being a function only of park space and population. Conceivably, then, a population unit in an area of high population density per unit of park space could have its recreation satisfaction scaled down even when little use was made of the area's parks. It seems apparent, then, that the demand (assignment) variables PEOPLE(I, J) should be weighted sums of recreation time expenditures, not population, i.e.,

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[17] The listing of TMALC has these factors multiplied tOgether instead of divided, as in manual # 11, p. 63. In addition, manual # 11, pp. 52-53, specifies that the denomenator should be multiplied by 250, whereas this is not done in TMALC. It is definitely warranted to properly limit PKUSE(J).

$$(PRKN(J, 1)/FLND(I))*(\sum_{K} TIME(I, K, 5, 2)*NMPU(I,K))$$

where TIME(I, K, 5, 2) is the recreation time obtained by population units of class K on parcel I and NMPU(I,K) is the number of those population units. Other constants of proportionality may have to be introduced to give comparable scales for supply and demand (e.g., population unit size, 500).

At the heart of this procedure are the concept of the "park" as the locus of all recreational activity and the restriction of assignment to an arbitrary grid around the residence. The Washington Study shows clearly that most leisure time is spent at home, 63% on weekdays and 56% on weekend days (see Table 2 below). Less than half of the out-of-home leisure time is spent on activities which utilize public or private facilities for recreation-type activities (the "other recreation" category in Table 2). That the average travel time on these recreation activities is only about ten minutes is deceiving, for that is the average over doers as well as non-doers and represents about 12% of the total time spent on the activities.[18] To tell whether or not an activity occurs close to home we need the travel time per occurrence, which unfortunately has not yet been computed in that study. Therefore, while we can say that "park" (or facility)-related activities involve less than one-fifth as much time as is spent in in-home leisure and in socializing in other residences, we do not know whether or not those park/facility activities take place in the neighborhood, as is assumed in the River Basin Model.

There is some question, then, as to the appropriateness of taking "park" use as a proxy for all social/recreational leisure behavior. This point will not be belabored further here, except to make two

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[18] In the Washington Study, the average travel time to movies was about forty minutes for participants, indicating a fair distance traveled.

TABLE 2: Relative Leisure Time Expenditures in the Washington, D.C. Metropolitan Area

Proportion of Leisure Time* Spent in ...	Weekdays	Weekends
the residence	.63	.56
out of the residence:**	.37	.44
... in socializing at other residences	.08	.09
... in education and training	.04	.01
... in part-time work	.02	.02
... in travel to full-time work	.10	.01
... in political, civic, organizational activities	.01	.01
... in religious activities	.01	.10
... in other recreation	.11	.20
Total:	1.00	1.00

* Categories have been grouped to be consistent with River Basin Model definitions.

** Travel times are included and the proportions obviously are calculated over non-participants as well as participants in activities, as for a population unit.

SOURCE: Hammer and Chapin, op. cit., pp. 50-54.

points. The first is that it is always possible to treat residential land as a kind of "park" in which visiting occurs. The capacity of such land for recreation could be a function of the number of residential space units provided there, which in turn depends on the mix of housing types and their levels of development. Capacity could be further specified by population unit class if assignments were made class-specific to represent within-class socializing. Given the number of people a space unit could serve in recreation (visiting), the capacities per % of parcel in various residential uses could be computed for comparison with such figures for parks (see manual # 11, p. 52). The second point to be made here is that if use of developed park space is thought of as including the patronizing of private facilities and establishments, a suitable method of pricing such use must be developed. It is true that there is a component to consumption expenditures in OPCM which is proportional to recreation time (see the next chapter). However, the prices charged depend on assignments to facilities that are part of the economic sector and have nothing to do with parks or their locations.

This brings us to the park use assignment procedure itself. In comparison to those procedures used in the model for employment and shopping (see OPCM, next chapter), park assignment is extremely crude. Its greatest inadequacy is that assignment of population units at I to a park J in its grid reflects the supply of park land at J but not the demand for that land. Since demand is unknown until assignments are made, it would seem that an iterative procedure is called for (as with shopping) unless some priority for ordering population units is established (as with employment).

A second problem is that assignments are made without regard to location; that is, there is no attenuation in assignment for the distances which would have to be traveled. A unit of park capacity is assumed to be just as attractive on the edge of the grid as it is on the adjacent parcel. The procedure may be realistic in its apportionment of demand to many opportunities rather than assignment to a sincle one (cf. OPCM), and there are gravity models and iterative balancing procedures which would serve this purpose well, allowing attenuation with distance and feedback effects of crowding (demand/supply relationships at "destinations," or parks) on individual demands (at "origins " or residences). Because such changes mean substantial modification of the assignment procedure, no further attempt will be made to spell out how they should be accomplished. Instead, we will turn to some more straightforward issues and suggested modifications.

It was suggested that demands and assignments for park use be carried out in units of recreation time rather than population, which means a revision in the composition (though not form) of the park use indices computed for parks and population units. There would be two ways to apply the revised park use indices. The first is simply to apply them in PKAL for dissatisfaction as is done at present. The second would be to use them to scale down recreational time expenditures until an acceptable use index level is reached whenever there is excessive overcrowding. Two examples are the treatment of public education time, which is reduced until demand no longer exceeds supply, and the treatment of residential densities in migration, where outmigration is forced until demand exceeds supply by no more than some "acceptable" percent. In the first case the use index would be brought to unity for the population unit, so that the use of the index in scaling dissatisfaction in recreation would no longer be necessary, the dissatisfaction coming directly from the reduction in recreation time and the possible increase in involuntary time.[19] In the second case, there could be all three effects with park overutilization: less recreation time, more involuntary time, and an altered scale factor for recreation time in computing dissatisfaction.

The recreation time reduction suggested here is not reassignment, in that there would be no redistribution of the excess demands at parks or alterations of assignments at parks having adequate capacity. To balance supply and demand at park J where there was overutilization, for example, the assignment PEOPLE(I,J) from each parcel I in the grid on J could be reduced by the amount

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[19] If the recreation time decrements were reassigned to parks having PKUSE < 1 (or MAXUSE), and the process were repeated until either all parks in I's grid had use indices not greater than one (MAXUSE) or I had no further decrements in recreation time, then we would have a simple reassignment procedure. Any time still unassigned at that point would become involuntary.

(1.0 - 1.0/PKUSE(J))*PEOPLE(I, J)

Total reduction in recreation time at I would be the sum of any such reductions at parks J in its grid, and involuntary time would be increased by this amount. If a permissible level of overcrowding were specified, say MAXUSE, then the deflation factor would be

(1.0 - MAXUSE/PKUSE(J))

Because all of these effects operate in the same direction to increase dissatisfaction, the manner of applying the use index may seem of minor importance. It is true that undeveloped park space is not likely to be regulated as to the extent of overutilization permitted, whereas educational facilities (and even developed park space, for that matter) might be. However, from a behavioral viewpoint the determination of recreation time should not be independent of park capacity. There is another reason for adjusting recreation time, and that has to do with the fact that recreation time is a factor in determining consumer expenditures for goods and services. By not reducing recreation time for overcrowding we may force the population unit to maintain an artificially high level of (recreation-based) consumption expenditure.

There is one last issue to be brought up with respect to the park use index PKAL(I) for a population unit on parcel I, however the park use indices PKUSE(J) for parks J are determined. With PKAL(I) being the minimum of the PKUSE(J) in the grid on I, it need not reflect the nature of the actual assignment of I's recreational demand. It would be entirely possible to have predominant assignment to one park in the grid where there was overcrowding, while the park use index for those population units is based on supply and demand at some less utilized park.[20] The most straightforward modification of PKAL(I) would be to define it as the weighted sum

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[20] The disparity represented by this effect is greatly reduced if there is reassignment, of course.

$$\mathrm{PKAL(I)} = \Big(\sum_{\substack{\mathrm{J} \\ \text{in grid} \\ \text{on I}}} \mathrm{PKUSE(J)*PEOPLE(I,\ J)}\Big)/\mathrm{TOTAL}$$

where TOTAL is the sum over J of the PEOPLE(I, J) assignments. Following earlier suggestions this would be equal to the total recreation time expenditures of population units at I.

In connection with the discussion of iterative assignment procedures the matter of transportation for the use of parks was brought up. It seems only reasonable that such transportation be identified and times and costs assessed the population unit. In iterative assignment based on travel it would be expected that the dollar values of time in leisure and travel would play roles similar to those in job and shopping[21] assignments. Short of the development of such assignment procedures for recreation, it should still be possible to make appropriate charges for travel. Travel time and cost to each park could be determined any of several ways, including the prior specification of route and/

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[21] At present, they have no role in shopping, but one is suggested in the next chapter.

or mode (such as in part-time work) and, at the other extreme, the use of VALTIM to pick the minimum shadow cost trip type. To produce consistent time budget expenditures the following alteration might be made in the assignment procedure: assign gross times to parks rather than net (non-travel) times. Everywhere in the River Basin Model travel times and costs to destinations are assumed proportional to the time spent there, on the implicit assumption of a given amount of time at the destination for each trip. Thus, if AT(I, J) is the travel time (however determined) to go from I to the park at J, the total travel time is AT(I, J)* PEOPLE(I, J), where PEOPLE(I, J) is the recreation time to be assigned to park J and is now

$$\begin{aligned}\text{PEOPLE}(I, J) = {} & (\text{PRKN}(J, I)/\text{FLND}(I)) \\ & *\Big(\sum_{K} \text{TIME}(I, K, 5, 2)*\text{NMPU}(I, K)\Big)/ \\ & (1 + \text{AT}(I, J))\end{aligned}$$

Obviously, the sum of I's travel and recreation components for park I equal the total recreation assignment under the non-travel procedure presently used (modified for time assignment rather than people). Now, however, the park use index for a park J, when computed by the same formula as before, will exclude the travel component. This will reduce actual utilization of parks at greater distances from population units. Total travel costs to J from I would be AC(I, J)*PEOPLE(I, J). The travel times and costs should be summed and charged against the appropriate budget for the population unit. The recreation expenditure finally determined would be TIME(I, K, 5, 2) decremented for the travel times. If population units attempt to optimize in allocating time, the optimal allocation to recreation would include an estimate of the travel component, analogous to the example presented earlier for part-time work. Consequently, the dollar valuations for time in leisure and travel would now reflect recreation travel parameters (and ultimately those for shopping, plus prices) as well as those for part-time work and wages. These complications to the optimization problem will be considered later.

Summary

This rather lengthy chapter has made numerous suggestions for modifying time and location assignment procedures for the Social Sector of the River Basin Model. Improved correspondence with reality has been the major objective. Admittedly, the achievement of that objective need not produce a "better" model in terms of its ability to simulate the whole system, of which the Social Sector is but one part, efficiently and at a reasonable cost. These suggestions are not meant to imply that the present Social Sector is thought inadequate. Quite to the contrary, the treatment of the Social Sector is exciting and capable of producing new insights into the interaction of households with "the system." Indeed, it is precisely because the implications are so provocative that so many issues arise in this view of behavior -- time use -- about which we know so little. These suggestions are offered, then, with full realization that the need for simplicity in such a pioneering model as this may preclude undertaking many of the modifications involved, some of which are rather picky attempts at consistency and comprehensiveness.

On the other hand, most modifications suggested in this report require the alteration of no more than several statements in the computer program, and it is felt that such changes would be well worth the effort involved.

SECTION X
MONETARY CONSUMPTION EXPENDITURES

The operations carried out thus far for a population unit in a round —migration, employment, and leisure activity — provide most of the information needed to construct a financial statement for the population unit. The remaining activity assignment to be made is that for shopping. On its completion sources of incomes and expenses for the round can be identified and the accounting procedures undertaken. Shopping assignment and financial bookkeeping methods will be reviewed in this chapter, and this review will tie up our examination of the detailed workings of the Social Sector as they pertain to time and money allocations and expenditures.

Consumer Expenditures

Assignment of population units to shopping establishments is made for the purchase of consumption goods and services. Before this assignment is attempted, the consumer demand for these goods and services must be determined. This is done in subprogram SETCOM, which computes consumer demand for a population unit as the sum of a base component which is fixed and a variable component which is a function of the recreation time obtained by the population unit. This demand, then, does serve to measure population unit inputs of goods and services to "recreation" (broadly defined); again, however, this is not made specific to the recreation facility itself, so that payments will not accrue to such facilities directly from population units. The fixed component varies by population unit class and reflects differences in their demographic composition and, possibly, differences in input mix to recreation by class. If recreation were dissagregated by type, it would be desirable to relax assumptions about differing input mixes and to let the model replicate such differences through preference/income/cost relationships.

For population units of type K on parcel I, demand for consumption goods (T = 1) or services (T = 2) is found in SETCOM as

IUN = MULT*(BAS + DIF*REN)

where BAS = BASCON(K, T) gives the fixed base consumption rate per population unit of class K for consumption of type T

REN = RENCON(K, T) gives the variable consumption rate per unit time in recreation per population unit for consumption of type T

DIF = TIME(I, K, 5, 2) is the recreation time obtained by a population unit of class K on parcel I

MULT = NUMPU(I, K) is the number of population units of class K on parcel I

In the current version of the model, the data are[1]

		BASCON		RENCON	
Class	(K)	T = 1	T = 2	T = 1	T = 2
low	(1)	21	7	.025	.000
middle	(2)	28	11	.050	.050
high	(3)	34	16	.100	.075

These data can be determined as

$$\text{BASCON}(K, T) = (22 - 9*T) + (14.5 - 6*T)*K + (-1.5 + T)*K*K$$

$$\text{RENCON}(K, T) = (.025)*((5 - 4T) + (-4.5 + 4T)*K + (1.5 - T)*K*K)$$

for T = 1, 2, and K = 1, 2, 3. From the table it is apparent that services are less sensitive to recreation time than are goods in an absolute sense, since their values on RENCON are lower:

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[1] These data are from manual # 4, p. 86. The source listing for SETCOM gives different values for RENCON.

$$\underset{T}{\Delta}\text{RENCON}(K, T) = 4*(K - 1) - K*K \leq 0$$

Similarly, the fixed consumption of services is lower:

$$\underset{T}{\Delta}\text{BASCON}(K, T) = -9 - 6K + K*K < 0$$

However, a greater proportion of the consumption of services is due to recreation than is true for goods for the upper two classes (but not the lowest, which uses no service in recreation):

$$\begin{aligned}&\underset{T}{\Delta}(\text{DIF}*\text{RENCON}(K, T)/\text{BASCON}(K, T))\\ &\quad =\Big(\text{DIF}/\text{BASCON}(K, 1)*\text{BASCON}(K, 2)\Big)\\ &\qquad *(.025)*(-43 + 19.5*K + 23.5*K*K\\ &\qquad\quad - 7.0*K*K*K)\\ &\quad > 0 \text{ for } K = 2, 3 \text{ but } < 0 \text{ for } K = 1\end{aligned}$$

Interestingly, consumption rates for goods plus services are simple linear functions of class. That is,

$$\sum_{T} \text{BASCON}(K, T) = 17 + 11*K$$

$$\sum_{T} \text{RENCON}(K, T) = (.025)*(-2 + 3*K)$$

The proportion of total consumption which is due to recreation time can be computed from this as approximately 1/10, 1/5, and 1/4 for K = 1, 2, 3, respectively, when DIF = 100, its upper limit under the time budget. These proportions all decrease as DIF drops below 100, but the order by class remains. Finally, we note that all consumption rates increase with class:

$$\Delta_K \mathrm{BASCON}(K, T) = (13.0 - 5*T) + (-3.0 + 2*T)*K > 0$$

$$\Delta_K \mathrm{RENCON}(K, T) = (.025)*(3*(T - 1) + (3 - 2*T)*K) > 0$$

However, the rates of increase vary by type. Fixed consumption increases at an increasing rate for services but at a decreasing rate for goods --

$$\Delta_K^2 \mathrm{BASCON}(K, T) = 2*T - 3 \quad \begin{matrix} < 0 \text{ for } T = 1 \\ > 0 \text{ for } T = 2 \end{matrix}$$

-- while the variable consumption rate increases at an increasing rate for goods but at a decreasing rate for services --

$$\Lambda_K^2\,\mathrm{REMCON}(K, T) = (.025)*(3 - 2*T) \begin{cases} > 0 \text{ for } T = 1 \\ < 0 \text{ for } T = 2 \end{cases}$$

Since there have been no studies for which comparable data on time and money expenditures for households have been collected and analyzed, it is not possible to make a direct evaluation of the validity of the trends assumed in SETCOM, although for all practical purposes they seem reasonable. The only qualification that might be made is one that has been made several times before, that it would be preferable to develop the model so that such trends would be output rather than input, thus providing an explanation for their origin in reality. For illustrative purposes, Table 3 below has been set up to give total consumption expenditures as a proportion of income under the default times for part-time work and recreation (manual # 4, p. 84), minimum and maximum limits on prices of consumer goods and services (50 and 150, respectively), and minimum and maximum limits on salaries by class, which can be summarized as

$$(11 - 5*K + 6*K*K)*100$$

and

$$(50 - 25*K*(K - 1))*100$$

TABLE 3: Hypothetical Consumption Expenditures As Proportions of Income

	Consumption Units		Expenditures as Proportion of Income			
			Low Price		High Price	
Class K	Total	Proportion Variable	Low Income	High Income	Low Income	High Income
1	28.50	.02	.85	.23	2.55	.61
2	40.00	.03	.62	.15	1.85	.46
3	51.75	.03	.43	.11	1.29	.32

respectively. That only 3% of consumption stems from recreation time in this example is disturbing, for it means that if these default time allocations are typical, recreation time has virtually no direct cost. Inclusion of transportation costs for recreation will help in this regard.

Shopping Assignments

The consumption levels determined by SETCOM are measured in "consumption units" for which a price per unit is charged at retail establishments. Establishments are differentiated as to the provision of goods or services, and population units are assigned to the different types of establishments separately. Obviously, consumption unit prices may differ by type of consumption as well as by establishment. The subprogram making the shopping assignments is OPCM. Since the procedure is the same for goods as for services, it will not be necessary to specify the type of consumption purchase involved.

The commercial optimizer, OPCM, attempts to assign population units to retail establishments to minimize the "shadow" cost per unit of consumption for the population unit. This shadow cost is a function of transportation cost, crowding at the establishment, price charged, and preference for shopping at the same establishment as in the previous round. Since crowding reflects interdependence in population unit assignments, the procedure is iterative and assignments are adjusted until certain conditions are satisfied. There are no time dimensions to this assignment procedure, even though the manuals (# 2, p. 121) claim that the crowding component is a proxy for excessive shopping time. Because the introduction of time elements will be advocated below, this procedure will be examined in more detail than was afforded the discussion of employment assignments.

In subprogram SETCAP the effective capacity of an establishment had been determined as

CAP = EM*BASE

where EM is the ratio of the number of workers hired to the number required for the given level of development, i.e., the "employment effect," and BASE is the base capacity as a function of the type (LUS) and level (LEV) of development and of the value ratio (VR) indicating maintenance and depreciation effects. Effective capacity will be the "supply" factor in demand/ supply relations for crowding. The first operation performed by OPCM is calculation of the least cost transportation route from each buyer (population unit) to each seller (establishment). As with part-time work, only automobile travel is allowed, but route choice is made with transportation subprogram TSCAN. It is unclear whether the dollar value of time parameter VALTIM is used in this choice. There is no reason why it should not be; indeed, it should be used, and its value should reflect shopping travel requirements if time allocations are to be consistent.

The next step is the computation of the "base perceived usage" of each establishment. These parameters are used by population units in their assessment of shopping opportunities, rather than actual usage levels, in order to minimize oscillations in assignments over iterations and to force a convergence such that few population units would choose to change establishments on further iteration. IDMDW, the base perceived usage of an establishment J on an iteration, is defined as a weighted sum of SERVE(J), its level on the previous iteration, and SERVNG(J), the actual level of usage at the end of that iteration. The functional relationship is

```
IDMDW = (SERVNG(J)*1000/IBANG
        + (1 - 1000/IBANG)*SERVE(J))
```

IBANG is an iteration parameter starting at a value of 1000 on the first iteration and incremented by 5% whenever an iteration seems to be diverging rather than converging on a solution. Over successive iterations actual usage has less weight and perceived usage more.

On an iteration a population unit I is assigned to that establishment which offers it the lowest "shadow cost" of shopping per consumption unit. The shadow cost of an establishment J is basically the money cost per consumption unit, being the price plus transportation cost, times a function of the ratio of the perceived usage to the establishment's capacity:

```
LSD = SHADOW*(PUC + LS)
```

where LSD is the shadow cost,

PCU = 100*CMWD(J, 2) with CMWD(J, 2) as the price of a consumption unit (in hundreds) at establishment J, LS = AC(I, J) is the transportation cost of the least-cost route via auto from I to J[2]

- SHADOW is a function of the ratio of perceived usage to capacity.

The demand/supply factor which is the argument to SHADOW is the variable RATIO:

RATIO = (IDMDW + CONSUM)/CAP

Here the population unit's own demand CONSUM is added to perceived usage if it did not shop here the last iteration; otherwise CONSUM is excluded from the calculation.[3] The function SHADOW of RATIO is

SHADOW = A1 + (RATIO - A2)*A3

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[2] It is unclear whether LS is just AC or AC + VALTIM*AT to include time valuation. The latter is more appropriate. In addition, it is implied that AC is of appropriate scale to represent the cost of however many trips are associated with one consumption unit's purchase. However, no such scale factor is identified.

[3] CONSUM is the variable IUN computed in SETCOM. It is stored in COMM(I, K, 3, T) where K is the class of this population unit on I and T = 1, 2 denotes consumption good (1) or service (2).

where the A's are themselves step functions of RATIO. SHADOW is something akin to a polynomial in RATIO, increasing with RATIO at an increasing rate in much of its range above .75. When RATIO $\leq$ .75, SHADOW is unity. Thus there is no crowding effect when RATIO $\leq$ .75.

Since perceived usage of each establishment tends to stabilize over successive iterations in the direction of the predominant trend of actual usage, RATIO and thus SHADOW also stabilize. Thus the chances increase that an establishment offering minimal shadow costs LSD to a population unit one iteration will also offer it minimal LSD the next. When most population units no longer switch establishments, iterations cease, and the population units are charged for the costs of their consumption purchases and transportation according to costs and prices for the establishments to which they were assigned. It should be noted that no charge is made for travel time, even though that variable may have been used in least-cost route selection if evaluation was made for the value of time via VALTIM. Again, such valuation and charges against time should be made. There is a problem of circularity, however, since total travel costs and times are functions of consumption --

AC*CONSUM and AT*CONSUM, respectively, -- and CONSUM is a function of recreation. The determination of recreation time may have exhausted the time stock, leaving none for this travel. On the other hand, travel time cannot be deducted from the budget prior to determining recreation time. This problem will be addressed below.

Although the iteration procedure used here appears to be an appropriate one, it could have been tied more meaningfully and directly to time utilization. In particular, even though SHADOW is supposed to be a proxy for shopping time effects (through crowding) no determination of shopping time nor charge for it against the time budget is made. Crowding implicitly forces a trade-off of money (for higher prices and/or travel costs) and thus an implicit dollar valuation of crowding (i.e., shopping time), despite the fact that VALTIM is a variable designed to do just that while being consistent with similar tradeoffs for other activities, and despite the fact that the population unit yields no time to shopping.

To show how crowding forces a valuation of shopping time, consider the following utility-maximizing problem based on the River Basin Model shopping assignment modified for time use and ignoring part-time work:[4]

$$\text{maximize} \quad U = \mathcal{U}(x, t_l, t_r, t_p, t_e)$$

$$\begin{array}{lll} \text{subject to} & 80S - px - gt_r \geq 0 & \text{(money)} \\ & 100 - t_r - t_s - t_l - t_p - t_e \geq 0 & \text{(time)} \\ & t_r - \alpha t_s \geq 0 & \text{(travel time)} \\ & t_s - cx \geq 0 & \text{(shopping time)} \\ & x - (at_l + b) \geq 0 & \text{(consumption goods and services)} \end{array}$$

where S is the full-time work wage rate and t_p and t_e are politican and education times.[5]

Here consumption goods and services x (purchased at price p) must not be less than a fixed amount b plus a variable amount at_l based on leisure ("recreation") time t_l, as in the River Basin Model. Two further relations have been added (it is proposed below that these be used in shopping assignment): travel time t_r cannot be less than some proportion α of shopping time t_s, and shopping time cannot be less than some proportion c of consumption level x. These relationships are analogous to relationships used previously in the River Basin Model. It will be assumed that these three constraints are definitional equations, so that their respective shodow prices, ψ_r, ψ_s, ψ_x, are non-zero.

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[4] Inclusion of all activities in a utility-maximizing model will be presented in the next chapter. See also the shopping model of Part One of this report and the part-time model re EMPRT.

[5] It is necessary to make t_p and t_e explicit here to avoid a trivial solution having no more variables than constraints.

First-order conditions for x, t_l, t_r and t_s can be solved to give the dollar values at the optimum of:

travel time:
$$\frac{\psi_r}{\lambda} = \beta + \frac{\theta}{\lambda} - \frac{1}{\lambda}\frac{\partial \mathcal{U}}{\partial t_r}$$

shopping time:
$$\frac{\psi_s}{\lambda} = \frac{\theta}{\lambda} + \alpha\,\frac{\psi_r}{\lambda}$$

consumption level:
$$\frac{\psi_x}{\lambda} = p + c\,\frac{\psi_s}{\lambda} - \frac{1}{\lambda}\frac{\partial \mathcal{U}}{\partial x}$$

Travel time valuation here reflects marginal travel costs β, the general dollar value of leisure time θ/λ, and the marginal utility of travel (which may be negative, thus increasing this valuation). The consumption constraint dollar value is that for shopping weighted by the crowding factor c, plus price and less (the negative of) the marginal rate of substitution of consumption for money. Parameter c represents crowding, for it determines how much shopping time is required for the purchase of a unit of goods and services. Shopping time valuation in dollars is the general dollar value of time plus the dollar value of time in travel weighted by the travel/shopping time factor α.

In choosing the optimal retail establishment, there will be first-order conditions for location variables l , where the travel parameters α and β and price p and crowding factor c are functions of such variables. Such a first order condition can be written to give an additional equation for the evaluation of shopping time in dollars as

$$\frac{\psi_s}{\lambda} = \frac{-1}{\partial c/\partial l}\left\{\frac{\partial p}{\partial l} + c\left(\alpha\frac{\partial \beta}{\partial l} + \frac{\partial \alpha}{\partial l}\left(\frac{\psi_r}{\lambda}\right)\right)\right\}$$

Finally, under the fiction of a continuous trip type variable m, optimal choice of route mode yields the condition

$$\frac{\psi_r}{\lambda} = -\alpha\frac{\partial \beta/\partial m}{\partial \alpha/\partial m}$$

relating the dollar valuation in travel time to the marginal changes in travel parameters. These two conditions will be brought up again below.

Next, let us revise the shopping assignment variables to reflect the time relationships developed above. Basic to this approach is the redefinition of variables in the time dimension. First, there is the modification of the function CAPCTY used in SETCOM to determine effective capacities CAP so that these capacities will be in customer shopping time units. As before, these effective capacities are "normal" design limits based on employment and condition of the establishment. Only a simple scaling should be necessary for this modification. Let actual usage of the establishment last iteration, SERVNG(J), and perceived usage last iteration, SERVE(J) also be in time units (see below), and let perceived usage <u>this</u> iteration be defined as before --

```
IDMDW = (SERVNG(J)*1000/IBANG
        + (1 - 1000/IBANG)*SERVE(J))
```

-- in terms of the iteration parameter IBANG. Then we have perceived utilization rate PRATIO as

$$PRATIO = (IDMDW + SHOP)/CAP$$

where the increment SHOP to perceived usage (perceived gross shopping time at the establishment) is the population units' own shopping time requirement and is added only if the population unit did not shop here the previous iteration.

PRATIO is now used to determine PTCU, the "perceived" shopping time per consumption unit, in the same manner as SHADOW was derived in the River Basin Model, as an increasing (step) function of PRATIO:

$$PTCU = B1 + (PRATIO - B2)*B3$$

As before, the B's may be functions of PRATIO.[6] Note, however, that the population unit shopping requirement SHOP is also "perceived", since it is the population unit's estimate of the time it will take to shop for its consumption demand, CONSUM, and therefore it is based on PTCU. Specifically,

$$SHOP = PTCU*CONSUM$$

By substitution of this relation, we get PTCU as

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[6] As pointed out previously, programming and analysis would be simplified with continuous rather than step functions.

$$\mathrm{PTCU} = (\mathrm{B1} + (\mathrm{IDMDW}/\mathrm{CAP} - \mathrm{B2})*\mathrm{B3})/(1 - \mathrm{CONSUM}*\mathrm{B3}/\mathrm{CAP})$$

Of course a new function for PTCU in terms of IDMDW, CAP and CONSUM could be specified; the old form was kept mainly for illustrative purposes.

It is proposed that shopping assignments now be made to minimize a unit cost function which gives explicit dollar valuation to time in shopping and travel and includes travel cost as well as the price of goods, and also retains the perceived crowding effect (now based on time) to stabilize the iteration procedure. The new "shadow cost," then, is

$$\mathrm{LSD} = (\mathrm{PCU} + \mathrm{AC}) + \mathrm{VALTIM}*(\mathrm{PTCU} + \mathrm{AT})$$

Assignment for a population unit at I would thus be to establishment (location) J for which LSD is minimal. As PTCU stabilized over iterations, so would choice of the optimal establishment. At the end of an iteration the actual amount of goods or services purchased at J, GCU, is the sum of the CONSUM for those I assigned there.

Since the time budgets of population units must be charged for actual shopping time as well as travel time (AT*SHOP),[7] the actual shopping time per consumption unit must be determined. The actual utilization rate for the establishment this iteration is

$$\text{ARATIO} = \text{GTIM}/\text{CAP}$$

where GTIM is the new gross utilization (and will be SERVNG(J) for the next iteration). It seems only reasonable that actual unit shopping time ATCU be the same function of ARATIO that PTCU is of PRATIO. That is, population units do not estimate shopping time incorrectly but only base such estimates on distorted perceptions. Moreover, since

$$\text{GTIM} = \sum_{\substack{\text{I shopping}\\ \text{at J}}} \left(\text{ATCU*CONSUM(I)}\right) = \text{ATCU*GCU}$$

ATCU can be computed as

$$\text{ATCU} = (\text{B1} - \text{B2*B3})/(1 - \text{B3*GCU/CAP})$$

once assignments are made. The shopping time for a population unit is thus ATCU*CONSUM, and this should be charged against the time budget.[8]

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[7] As explained before, it is assumed that AC and AT are scaled to represent levels per trip times the (constant) number of trips to be made per consumption unit bought.

Under such a procedure as this the change in shadow cost ISD in switching from the optimal establishment to another would be

$$\Delta \text{ISD} = (\Delta \text{PCU} + \Delta \text{AC}) + \text{VALTIM}*(\Delta \text{PTCU} + \Delta \text{AT}) > 0$$

This means that at the optimum

$$-\left(\frac{\Delta \text{PCU} + \Delta \text{AC}}{\Delta \text{PTCU} + \Delta \text{AT}}\right) \begin{cases} < \text{VALTIM if } (\Delta \text{PTCU} + \Delta \text{AT}) > 0 \\ > \text{VALTIM otherwise} \end{cases}$$

This means that if the change brings a net increase in travel and shopping time, then any dollar savings which might arise will be less than the dollar value of the extra time. If the change brings a net reduction in travel and shopping time, then there will be an increase in costs which exceeds the dollar value of those time savings. Manipulations of the shadow price relations in the utility-maximizing example considered above suggest that VALTIM reflect both the shadow price of shopping and the shadow price for leisure time in general.[9]

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[8] In addition, EMPTAX should be called to compute automobile and sales taxes.

One problem which has been overlooked here is that shopping assignments are made after all time budget determinations have been completed. Specifically, there may not be enough time left in the budget (involuntary time) to cover the shopping and travel times required for the consumption that recreation time con-

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[9] After substitution for ψ_r/λ we have

$$\frac{\psi_s}{\lambda} + \frac{c}{\alpha}\cdot\frac{\partial\alpha}{\partial l}\left(\frac{\theta}{\lambda}\right) = -\left(\frac{\frac{\partial P}{\partial l} + \alpha c\,\frac{\partial\beta}{\partial l}}{\frac{\partial c}{\partial l} + \frac{c}{\alpha}\,\frac{\partial\alpha}{\partial l}}\right)$$

which is comparable to

$$\text{VALTIM} \simeq -\,\frac{\Delta\text{PCU} + \Delta\text{AC}}{\Delta\text{PTCU} + \Delta\text{AT}}$$

for changes in location (establishment). For trip choice we have

$$\frac{\psi_r}{\lambda} = -\,\alpha\left(\frac{\partial\beta/\partial m}{\partial\alpha/\partial m}\right) \quad \text{versus} \quad \text{VALTIM} \qquad \frac{\Delta\text{AC}}{\Delta\text{AT}}\ .$$

tributes to.[10] There are several ways to balance the time budget. One is to reverse the time assignment priorities and take whatever time is necessary away from, first, political activity and then education. No modifications other than the proper decrementing of those amounts is required. If this does not create enough time, part-time work increments of 10 units could be freed, the last assigned taken first, until the time budget balanced. This would change the employment situation, of course, and may not be desired for that reason. On the other hand, the "quitting" of a job is not necessarily unrealistic. Modifying the employment lists is a simple matter.

A second method would be to make assignments first only on the basis of fixed consumption components. This is likely to take care of as much as three-quarters of the consumption unit assignment and could be done before other time assignments are made, with proper charges for travel and shopping time. Here there are several additional options. After other time assignments are made, the assignment of variable consumption based on recreation time could be made independent of the first assignment, by another run of the procedure with time budget balancing carried out as in the first alternative above. Or there could be a second run of the "scaled-down" or "simultaneous" varieties suggested below. These second-runs could utilize usage levels carried over from the first run for fixed composition but would allow choice of a second establishment as well. Finally, the second run could be foregone, and variable consumption assigned (after recreation time determination) to the same establishment as the first run (fixed consumption), using total utilization rates to determine the crowding effect on shopping time. Once again, time budget balancing could be carried out by the first method above or by the "scale-down" procedure below. Note that here the time changes forced upon other activities would be very much smaller than in the first method for total consumption, since most travel for shopping and shopping itself would already have been absorbed.

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[10] Obviously, if there is enough involuntary time reserved for these, it can be decremented by their amounts.

The third method is a "scale-down" method. All population units having time budget violations after shopping assignment would have their recreation times reduced until their budgets balanced. Such reduction would reduce travel and shopping along with recreation, so that the total reduction required of recreation would be less than the amount by which the total stock of time (100) was exceeded. Since variable consumption is such a small portion of the total, this may not balance the time budget even when no recreation time remains. Another way to scale down would be to reduce other leisure activities (politics or education or both) at the same rate as recreation until the budget balanced.

Finally, recreation alone could be scaled down and, if exhausted, the first method above applied to the remaining activities. In such scaling, utilization at the establishment would drop, thus making shopping more efficient. If desired the recreation times of other population units could be scaled *upwards* accordingly, subject to their time budget limitations.

The last method is to develop a "simultaneous" solution, whereby recreation time could be modified as shopping assignment iterations proceed. Any of the methods above could be used to balance the time budget for a population unit within an iteration upon completion of its shopping assignment for that iteration. In this manner the consumption variables themselves could fluctuate from iteration to iteration but would converge on stable levels as the iteration process itself converged. A last alternative would be a deterministic simultaneous solution, as follows. There are the relations

$$\begin{aligned} \text{SHOP} &= \text{CONSUM*PTCU} \\ \text{CONSUM} &= \text{BASCON+ RECON*TIME}(5,\ 2) \\ \text{TRAV} &= \text{AT*SHOP} \\ \text{SHOP} + \text{TRAV} + \text{TIME}(5,\ 2) &\leq \text{REMTIM} \end{aligned}$$

before the final level of recreation time TIME(5, 2) is set in TMALC, plus the equation for PTCU as a function of PRATIO, or more exactly, of IDMDW and CAP (which are both constant for the iteration) and CONSUM. This is a set of five equations and five unknowns: PTCU, TIME(5, 2), SHOP, CONSUM, AND TRAV. If REMTIM is large enough, then, there is a deterministic solution for PTCU for an establishment J for which the time budget will be balanced. A different recreation, shopping and travel time would be computed for each establishment.[11] Assignment to that one which minimizes the shadow cost would also produce the optimal time set. The iteration procedure itself could be carried out as before. If REMTIM cannot handle SHOP plus TRAV even when recreation time is zero, a decrement variable could be created to keep track of the amount of the disparity. When iteration ceases, any population unit with such a decrement at its final shopping assignment would have that decrement charged against its political time, a simple bookkeeping matter.

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[11] At present that value of PTCU is one of the roots of a quadratic equation. A simplified formula could be developed if desired.

Note that choice of a time-budget balancing method means possible integration of parts of TMALC with parts of SETCOM and OPCM, and a reconsideration of the time priorities assumed in TMALC. It should be emphasized that the description of these changes is much more complicated than the making of the changes themselves. For example, the modified assignment procedure with simultaneous deterministic solution of recreation time could be inserted with a change of about fifty programming statements.

Financial Accounting for Population Units

For completeness, the accounting equations for deriving monetary expenditures are listed here, with comments only if some change should be made to make them consistent with the modifications in time assignment procedures suggested previously. It should be remembered that all population units of a class on a parcel are assumed to make the same time allocations and decisions, and thus while time assignments are usually made for population units singly, the final time and money accounts present expenditures for the population units of a class on a parcel as a group as well as "per capita." Many of these computations are carried out in the summary subprograms WRYOU and PRINTY, but others are performed elsewhere. The relevant subprogram name is enclosed in parentheses. The residential parcel in question, denoted I, will not be an explicit subscript unless necessary. K denotes class of the population units and J a location.

1. Cash transfers (CASHT)

```
    ITEST = CSBL(I, K) + MISI(I, K)
```

New cash balance ITEST is the old one, CSBL, plus net transfers ("miscellaneous income") MISI

2. rent (DEPREC)

RNPD = NP*RN*BASE(K)/BASE(1)
NP = RPOP(I, K) number of population units
RN = RENT(I) rent per space unit
BASE(K) space consumption parameter

3. health (WRYOU)[12]

HLTCST = BASE(K) + MINO(C, 100)*ONE(K)
+ MAXO(C - 100, 0)*TWO(K)
C = COLF(I) coliform count
BASE(K) = 1000*2**I (data)
ONE(K) = BASE(K)/50 (data)
TWO(K) = BASE(K)/20 (data)

HLTCST reflects only part of illness time, since the latter, as proportional to Health Index HLIN, contains arguments other than COLF.

4. income for full-time work (EMP)

$$\text{SALSUM} = \sum_{J} \text{SAL(J, K)*NWK(I, J, K)}$$

SAL(J, K) salary paid class K at job J
NWK(I, J, K) number persons working at J = WINCLS(K)*NP(I, J, K)
WINCLS(K) labor force for class K (data)
NP(I, J, K) number of population units of class K and I working at J

---

[12] There is some question as to whether or not health cost is actually charged against the money budget.

5. income taxes for full-time work (EMPTAX)

$$\text{TAXSUM} = \text{SALSUM}*(\sum_{J} \text{RAT}(J))$$

RAT(J) tax rate of appropriate type or jurisdiction J (including automobile tax)

6. transportation cost for tull-time work (EMP)

$$\text{AUCSUM} = \sum_{J} \text{AUC}(J)*\text{NWK}(I, J, K)$$

AUC(J) = AC(I, J, L) the cost of going from I to J by least shadow cost mode L

7. income from part-time work (EMPRT)

$$\text{BASET}(1) = \sum_{L}\sum_{J} \text{PERCNT}*\text{BSTSAL}(L, J, K)*\text{NPN}(I, J, K, L)$$

BSTSAL(L, J, K) salary of part-time job J assigned on the Lth iteration

PERCNT = .125*WINCLS(K) scales salary to equivalent of one-eighth job for 10 time unit increment per iteration per worker

NPN(I, J, K, L) number of population units assigned to part-time job J on iteration L

Note that no correction is made if last job assignment for a population unit involves time less than ten units. Allocation could be constrained to multiples of ten.

8. income tax for part-time work (EMPTAX)

$$\text{TAXSUM}(1) = \sum_{L}\sum_{J} \text{PERCNT}*\text{BSTSAL}(L, J, K)*\text{NPN}(I, J, K, L)*\text{RAT}(J, L)$$

Excludes automobile tax.

9. transportation cost for full-time work (EMPRT)

$$BASET(2) = \sum_{L}\sum_{J} PERCNT*BSTCST(L, J)*NPN(I, J, K, L)$$

BSTCST(L, J) = AC(I, J, 1) the cost of going from I to J (for job of iteration L) via automobile by least shadow cost route

10. automobile tax for part-time work

$$TAXSUM(2) = \sum_{L}\sum_{J} PERCNT*BSTCST(L, J)* NPN(I, J, K, L)*RAT(J)$$

Unlike auto tax for full-time work, which is based on income, this tax is based on transportation cost.

11. welfare (EMP)

WELFAR = UN*WINCLS(I)*BUFF(30)

UN = UNEM(I, K) number of unemployed population units of class K

BUFF(30) unemployment compensation in I's jurisdiction

12. adult education (private; public is free) (WRYOU)

EDCOST(1) = NP*TIME(I, K, 3, 2)*3000

Note absence of transportation cost and (auto) tax. These could be

TRVED(L) = NP*TIME(I, K, L, 2)*AC(I, L)

EDTAX(L) = NP*TIME(I, K, L, 2)*RAT

AC(L) is the travel cost by auto from I to education facility L for the jurisdiction (L = 2 public, L = 3 private) by the least shadow cost route, scaled to be per time unit as for work.[13]

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[13] Travel times and costs in the River Basin Model are used as given for full-time work. Therefore, per unit time in other activities they can be scaled by the factor 1/80.

13. children's education (private; public is free) (LOADSC)

EDCOST(2) = SCOUT*CPED(K)

CPED(K) cost of private education per pupil for class K: (125, 175, 300) = 150 + 25*I*(3*I -5)/2

SCOUT number of children in private school = NP*P1TOP(K)*(1 - AMT(J, K))

P1TOP(K) number of school children for a population unit of class K

AMT(J, K) proportion of children of class K which can be assigned to the public school of jurisdiction J as a function of facility value-ratio, teacher skills (class in employment), and pupil teacher ratio

This expenditure can be altered by the population unit only through collective influence on the Public Sector (political activity) to change AMT. All lower class children are assigned to public education.

14. Political activity: no cost (a travel component could be determined).

15. Consumption Expenditures (see discussion of SETCOM and OPCM): these are stored as COMM(I, K, 3, L) for L = 1 (goods) and L = 2 (services) gross for the K class at I.

16. Sales tax for consumption (EMPTAX)

SLTX(L) = COMM(I, K, 3, L)*RAT(L) L = 1,2

17. Transportation costs for consumption (OPCM)

TRCOM(L) = COMM(I, K, 3, L)*AC(I, J) L = 1,2

J is the establishment location for good service L. Travel and shopping times should be added to the present model. Travel time and cost should be based on shopping time instead of consumption expenditures.

18. Automobile Tax for consumption (shopping) (EMPTAX); at present this is computed like sales tax, but it should be based on shopping time (transportation time).

Subprogram WRYOU takes these expenditures for population units of a class K on a parcel I and derives the cash balance (net savings) on a per population unit basis. These are averages for the class on the parcel and will cover population units in various circumstances (e.g., employed and unemployed) even though all made the same original time allocations. Total income is averaged as TEM. Rent, travel, consumption, education and taxes are averaged to give expenses TEMPP. The new cash balance, then, is

$$SAV = TEM \quad TEMPP + CSBL(I, K)$$

and is stored as the new value of CSBL(I, K), where the old value in this equation has been adjusted for cast transfers this round.

SECTION XI

SOME CONCLUDING THOUGHTS ON SIMULATION

This report has reviewed the literature on utility theory for major activity time-oriented developments, and it has analyzed the time-related assignment procedures of the River Basin Model with reference to those developments. It is the conclusion of this report that the role of time use in Social Sector behavior is of fundamental importance and that this aspect of the River Basin Model makes it unique in its ability to represent in a modelling context the crucial dimensions of decisions of real-world households. This report was not charged with the actual development of simulation procedures for time allocation decisions, only with the analysis of the manner in which those decisions are translated into time and money expenditures in the model. It is felt that this analysis has been thorough and that the basic structure of Social Sector operations is appropriate for its primary task of inter-relating the behavior of households with the behaviors of institutions and agencies in other sectors, all in the delicate balance between human and non-human environmental systems.

The numerous modifications that have been suggested to make activity assignments consistent and comprehensive all lie well within the present structure of the model, and it is possible to carry out these modifications with a minimum of reprogramming of the model. Indeed, it has been demonstrated that with these modifications the model's structure for activity assignment embodies processes in which the static optimizing behavior of households (population units) is modified by the uncertainties and feedbacks of a dynamic environment. It seems reasonable, therefore, that the rules by which allocational decisions of population units will be made in simulation should be derived from the static optimization theory. The dynamism of the assignment procedures (subprograms) means that such allocations will always be before the fact, since they are based on knowledge of the past and only estimates of the present and future, so that the expenditures assigned are always likely to be at least slightly out of equilibrium for the population unit. From round to round the population unit seeks to modify its allocations in response to a changing environment, but it is never able to completely anticipate all the changes that are forthcoming.

In the interest of facilitating the development of allocational rules for a simulation subprogram to replace the time allocation input subprogram TALOC, a Social Sector-oriented optimization problem will be presented below. Given some method by which the population unit estimates the prices and costs and other operating parameters of the system for the round,[1] the solution of the first-order conditions for the optimization problem will give relations to be used as rules for population unit allocation of the leisure time stock and determination of dollar values of time for input to the assignment subprograms. The treatment here is somewhat superficial, for problems remain in the simulation of inter-unit decisions, such as boycotts and political pressure and voting, not to mention reconsideration of the nature of the objective function (dissatisfaction index, or Quality of Life Index) and other behavioral assumptions which have been accepted below without comment. These include proportionality assumptions between certain activity times and between activity times and costs, reflecting as they do implicit assumptions concerning activity frequencies and the average time and cost of each episode. It would be desirable to make such relations explicit and to compare their implied values against empirical findings, as might be derived from the data on the Washington Study. At present, those data are not in the proper form for comparisons other than those presented elsewhere in this report, however.

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[1] A simple method would be to modify the actual parameters of the last round for trend effects or for the effects of outcomes to the appropriate institutions.

Static Optimization for Population Units

The analysis to be presented here gives an example of the decisions a population unit might make to maximize satisfaction as it perceives the state of the system. It is necessary to distinguish, then, between the values of parameters used here, based as they are on perceptions, and the actual values to be used in assignment procedures. The point is that the best strategy for the population unit *is* to allocate to maximize satisfaction according to the state of the system as it sees it, with possible modification for probabilistic distributions for the likelihoods of different states (not considered below). The time and money budget constraints reflect the accounting equations of TMALC and WRYOU, respectively, and will be used as presented there without elaboration. To simplify various income tax rates have been aggregated to single parameters for full and part-time work; they are specific to job location and residence since some components could vary by those locations. Automobile tax rates have also been presented as specific to location, although this may not be necessary. Rent, health and child education expenditures have been aggregated to the variable M, which does not vary with time allocations. All time uses are explicit in the time budget. Two of those are fixed, full time work at 80 and illness as proportional to HLIN. The total time stock, then, will be 180 units to allow the possibility of using work time for leisure in the event of unemployment.

In a first look at the optimization problem the objective function will be a general utility function but one which is separable as to the effects of savings, travel, recreation, and involuntary time as a group versus the effects of education, political activity, and cash transfers as a group. That is, utility U will be the sum of the function $\mathcal{U}$ of the first group of arguments and the function $\mathfrak{J}$ of the second group. In this form $\mathcal{U}$ is a generalized time index and $\mathfrak{J}$ is a generalized composite population, neighborhood and health index. The separation of arguments in this manner is meant to indicate that utility is affected directly through $\mathcal{U}$ but that $\mathcal{G}$ is only a representation of the present value of lagged effects of its arguments on the future state of the system and the population unit's position relative to it. Savings level has been included in $\mathcal{U}$ because without it or without a limit to dissaving (also included as a constraint) the money budget will have little effect. Additional constraints on the composition of the travel time aggregate and the derivation of shopping time are left explicit rather than removed by substitution because explicit shadow prices for them are desired. Travel is not differentiated by purpose in $\mathcal{U}$. The optimization problem here, then, is a composite of the ones presented earlier in connection with EMPRT and OPCM, although consumption expenditures x have been omitted from the utility function here and from the money budget by substitution with a function of recreation time, in keeping with the River Basin Model. For further explanation of the relationships included in the problem, see the previous discussions of the River Basin Model assignment procedures.

The optimization problem for the population unit becomes maximize $U = \mathcal{U}(t_5, t_6\ t_7, V) + \mathcal{G}(t_2, t_3, t_4, k_j)$

subject to

the time budget (shadow price θ):

$$180 - \sum_{i=0}^{9} t_i = 0$$

where t_0 = 80 full-time work;
t_1 = part-time work;
t_2 = adult public education;
t_3 = adult private education;
t_4 = political time;
t_5 = recreation;
t_6 = transportation;
t_7 = shopping (goods);
t_8 = shopping (services);
t_9 = illness;
t_{10} = involuntary.

The money budget (shadow price λ):

(income) $S_o 80 + S_1 t_1 - \sum_j k_j$

(income taxes) $- r_o S_o 80 - r_1 S_1 t_1$

(auto taxes) $$- \sum_{\substack{i=1\\ i\neq 6}}^{8} r_{ai}\, \beta_i t_i$$

(expenditures) $$- \sum_{\substack{i=0\\ i\neq 6}}^{8} \beta_i t_i - \sum_{i=7}^{8} (a_i + b_i t_5) P_i - M$$

(sales taxes) $$- \sum_{i=7}^{8} (a_i + b_i t_5) r_i$$

(net savings) $$- V$$

$$= 0$$

travel time (shadow price ψ_r):

$$t_6 - \sum_{\substack{i=0\\ i\neq 6}}^{8} \alpha_i t_i = 0$$

shopping times (shadow prices ψ_{si}, i = 7, 8)

$$t_i - c_i(\alpha_i + b_i t_5) = 0$$

limit to dissavings (shadow price ψ_v):

$$V + Z \geq 0 \quad \text{where } Z \geq 0$$

To simplify, the dissavings limitation constraint can be removed by substituting $\overline{V} = V + Z$, or $V = \overline{V} - Z$, in the money budget and in $\mathcal{U}$. $\overline{V}$ is then the amount by which the dissavings limit Z exceeds dissavings.

In addition to the constraints there are the following first-order conditions for time, savings and cash transfer allocations:

$$- \theta + \lambda \left\{ S_1(1 - r_1) - \beta_1(1 + r_{a1}) \right\} - \alpha_1 \Psi_r \leq 0 \quad \begin{cases} < 0 \text{ if } t_1 = 0 \\ = 0 \text{ if } t_1 \geq 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_i} - \theta - \lambda \beta_i (1 + r_{ai}) - \alpha_i \Psi_r \leq 0 \quad \begin{cases} < 0 \text{ if } t_i = 0 \\ = 0 \text{ if } t_i \geq 0 \end{cases} \quad i = 2, 3$$

$$\frac{\partial \mathcal{U}}{\partial t_4} - \theta - \lambda \left\{ \beta_4(1 + r_{24}) + 3000 \right\} - \alpha_4 \Psi_r \leq 0 \quad \begin{cases} < 0 \text{ if } t_4 = 0 \\ = 0 \text{ if } t_4 \geq 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_5} - \theta - \lambda \left\{ \beta_5(1 + r_{a5}) + \sum_{i=7}^{8} b_i(p_i + r_i) \right\} - \alpha_5 \Psi_r - \sum_{i=7}^{8} c_i b_i \Psi_{si} \leq 0 \quad \begin{cases} < 0 \text{ if } t_5 = 0 \\ \geq 0 \text{ if } t_5 \geq 0 \end{cases}$$

$$\frac{\partial \mathcal{U}}{\partial t_6} - \theta + \Psi_r \leq 0 \quad \begin{cases} < 0 \text{ if } t_6 = 0 \\ = 0 \text{ if } t_6 \geq 0 \end{cases}$$

$$- \theta - \lambda\beta_i(1 + r_{ai}) - \psi_r\alpha_i + \psi_{si} \le 0 \quad \begin{cases} < 0 \text{ if } t_i = 0 \\ = 0 \text{ if } t_i \ge 0 \end{cases} \quad i = 7, 8$$

$$\frac{\partial \mathcal{U}}{\partial t_{10}} - \theta \le 0 \qquad \begin{cases} < 0 \text{ if } t_{10} = 0 \\ = 0 \text{ if } t_{10} \ge 0 \end{cases}$$

$$- \frac{\partial \mathcal{J}}{\partial k_j} - \lambda \le 0 \qquad \begin{cases} < 0 \text{ if } k_j = 0 \\ = 0 \text{ if } k_j \leqq 0 \end{cases}$$[2]

$$\frac{\partial \mathcal{U}}{\partial \bar{V}} - \lambda \le 0 \qquad \begin{cases} < \text{ if } \bar{V} = 0 \\ = 0 \text{ if } \bar{V} \ge 0 \end{cases}$$

It is quite obvious that unless involuntary time t_{10} yields positive marginal utility (which it does not in the River Basin Model) no time will be allocated to it. The same applies to (dis-) savings $\bar{V}$ and to cash transfers when the change in their level is negative, meaning increased outflow or reduced net inflow. These relations are based on the assumption that θ and λ are both positive. Since the constraints are equations, they could take on negative values, but that is not likely. ψ_r can also take either sign, but disutility to travel is likely to assure that it will be positive. If it is positive then the marginal utility of education or politics would have to be positive for the allocation of time to that activity, and the shadow price on shopping would have to be positive as well if shopping occurs. That is reasonable, since shopping, like travel, uses time without adding utility.[3] Finally, with all shadow prices positive, the marginal utility of recreation would have to be positive for time to be allocated to it. These signs are necessary but not sufficient conditions, of course.

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[2] Only cash transfers from the population unit ($k_j < 0$) are choice variables. $-\partial\mathcal{J}/\partial k_j$ represents an increase in the outward transfer.

From these first-order conditions dollar valuations for time in various activities can be derived. That for travel is related directly to its marginal utility and the general dollar value of the time in leisure it preempts:

$$\frac{\psi_r}{\lambda} \leq \left(\frac{\theta}{\lambda}\right) - \frac{1}{\lambda}\frac{\partial \mathcal{U}}{\partial t_6}$$

---

[3] This version follows the River Basin Model in omitting shopping from the utility function. However, it might be desirable to treat it like travel, creating dissatisfaction.

Travel is practically mandatory, so that this will be an equation. The dollar value of time in shopping of type i = 7, 8 is

$$\frac{\Psi_{si}}{\lambda} \leq \left(\frac{\theta}{\lambda}\right) + \alpha_i \left(\frac{\Psi_r}{\lambda}\right) + (1 + r_{ai})\beta_i$$

This is the sum of the value of the time it takes from leisure, the value of the time it requires of travel, and the direct travel cost it creates, including taxes. Since there is a consumption component independent of time, this too will be an equation. For the dollar value of time in leisure in general,

$$\left(\frac{\theta}{\lambda}\right) \geq \left\{S_1(1 - r_1) - \beta_1(1 + r_{a1})\right\} - \alpha_1\left(\frac{\Psi_r}{\lambda}\right)$$

where the right hand side is the net part-time salary (wage) after travel and taxes and after deduction of the dollar value of the time spent in travel. If that net wage is great enough to equal the dollar value of time in leisure, there will be part-time work.

In choice of travel mode or route for an activity i, there is the first-order condition equating

$$-(1 + r_{ai})\frac{\partial\beta_i/\partial m_i}{\partial\alpha_i/\partial m_i} = \left(\frac{\Psi_r}{\lambda}\right)$$

where m_i is the fictitious continuous mode variable for activity i. In the choice of optimal locations for activity i, through the proxy location variable l_i, there are the following conditions:[4]

work (i = 0, 1), where by convention $r_{ao} = \frac{\partial r_{ao}}{\partial l_o} = 0$:

$$\lambda \left[\frac{\partial s_i}{\partial l_i}(1 - r_i) - S_i \frac{\partial r_i}{\partial l_i} - (1 - r_{ai}) \frac{\partial \theta_i}{\partial l_i} - \theta_i \frac{\partial r_{ai}}{\partial l_i} \right]$$

$$- \Psi_r \frac{\partial \alpha_i}{\partial l_i} = 0$$

shopping (i = 7, 8) :

$$-\lambda \left[(1 + r_{ai}) t_i \frac{\partial \theta_i}{\partial l_i} + (a_i + b_i t_5)\left(\frac{\partial p_i}{\partial l_i} + \frac{\partial r_i}{\partial l_i} \right) \right]$$

$$- \Psi_r t_i \frac{\partial \alpha_i}{\partial l_i} - \Psi_{si}(a_i + b_i t_5) \frac{\partial c_i}{\partial l_i} = 0$$

---

[4] If allocations are spatially static-- that is, the population unit does not consider locational changes or anticipate any that might occur--then these conditions are extraneous to a simulation procedure for population unit decision-making. On the other hand, rates of change with location could be approximated by use of wage, cost, and price differences utilized in the actual assignment procedures of the previous round.

where $t_i = c_i x = c_i(a_i + b_i t_5)$ as a function of consumption x and thus of recreation t_5, so that this is

$$- \lambda \left[(1 + r_{ai}) c_i \frac{\partial \beta_i}{\partial l_i} + \left(\frac{\partial p_i}{\partial l_i} + \frac{\partial r_i}{\partial l_i} \right) \right]$$

$$- c_i \psi_r \frac{\partial \alpha_i}{\partial l_i} - \psi_{si} \frac{\partial c_i}{\partial l_i} = 0$$

There could also be a "location equation" for the residence in connection with voluntary migration (see Chapter IV and MIGRAT):

$$\frac{\partial \vartheta}{\partial l} - \left[\sum_{\substack{i=0 \\ i \neq 6}}^{8} t_i \left((1 + r_{ai}) \frac{\partial \beta_i}{\partial l} + \beta_i \frac{\partial r_{ai}}{\partial l} \right) + \frac{\partial M}{\partial l} \right]$$

$$- \psi \left(\sum_{\substack{i=0 \\ i \neq 6}}^{8} t_i \frac{\partial \alpha_i}{\partial l} \right) = 0$$

Here $\partial \vartheta / \partial l$ represents the change in the value (in dissatisfaction) of the residential environment in terms of price and cost changes as well as amenities (see the definition of the indices in HSDSSW), and $\partial M / \partial l$ also reflects changes in expenditure parameters, especially rent.

Recreation activity is distributed to locations in a pro-rated fashion rather than in an optimizing fashion.

Its travel time and cost here are average values based on relative capacities and demands at parks, since they form the basis for pro-rating the recreation time to locations (see TMALC). If the pro-rating is to be done as part of this optimizing process, then t_5 would have to be disaggregated to t_{5j} for recreation times at parks j. In place of the first order condition for t_5 there would be a set of conditions, one for each t_{5j}, with travel cost β_{5j} and time α_{5j} coefficients specific to parks. If the partials $\partial\mathcal{U}/\partial t_{5j}$ were functions of the park demands and capacities, then the formula for pro-rating t_5 would be a property of the optimal solution. Education and political activity locations are fixed.

Allocations representing a static optimum are not likely to lead to an actual optimum, of course. For example, there is obvious interdependence between allocations, shadow prices, and activity locations; yet in making allocations the population unit does not know for sure where its activities will be assigned, since it does not know the exact levels of system parameters or competing allocations of other population units for the round. Therefore, shadow prices based on allocations for an assumed set of parameter values may actually lead to assignments to other locations with different parameter values which cause the associated time allocations to be non-optimal. Over rounds the system may stabilize, so that parameters change little and the population unit shadow prices and time allocations converge on values which minimize the chances for location changes. Of course, the opposite could happen, the system exploding because wildly fluctuating system parameters throw shadow price estimates for population units into confusion, precipitating radical location reassignment and extreme disequilibrium of time allocations. These dynamic properties are important in studying the behavior of the system, and it is felt that the static, short-term optimization of population unit decisions is appropriate here, for it creates a situation in which the population unit is continually making lagged readjustments in its behavior in the face of a constantly changing environment. This seems very much to be the real-world circumstance for households.

Given the population units guesses of the "best" or "most likely" locations of its activities and thus of the parameter values which are relevant to them, the population unit would use these to set the dollar values of time and make the time allocations. As described above, there will be different values of VALTIM to be used in different procedures. In fact, the objective function of each assignment procedure indicates exactly which shadow prices should be used in the time valuations. In work assignments the objective is to maximize the net wage after travel with a valuation to travel time included. That is, in EMP and EMPRT

$$\text{Maximize} \quad SAL - (AC + VALTIM(I)*AT)$$

for I = 0,1 for full- and part-time work. In the derivations above it has been shown that this should be corrected for income and automobile taxes r_i and r_{ai}.

$$\text{Maximize} \quad (1 - r_i)SAL - ((1 + r_{ai})*AC + VALTIM(I)*AT)$$

In this context the proper time valuation is

$$VALTIM(I) = \frac{\psi_r}{\lambda} \qquad I = 0, 1$$

the dollar value of time in travel as determined by the first order conditions to be consistent with (θ/λ) and the entire set of time and money allocations.

For shopping activities the objective would be written as[5]

$$\text{Minimize} \quad PCU(I) - (AC + VALTIM(I, 1)*AT + VALTIM(I, 2))*TCU(I))$$

where travel parameters are per unit of shopping time and TCU(I) gives the shopping time coefficient for purchases of type I = 7 (goods), 8 (services). With modification for sales and automobile taxes this is

$$\text{Minimize} \quad (1 + r_i)*PCU(I) - ((1 + r_{ai})*AC + VALTIM(I,1)*AT + VALTIM(I, 2))*TCU(I))$$

The valuation of travel time for shopping is the same as for work --

$$VALTIM(I, 1) = \frac{\psi_r}{\lambda}$$

-- while the valuation of shopping time itself is its shadow price in dollars

$$VALTIM(I, 2) = \frac{\psi_{si}}{\lambda}$$

---

[5] This is a change in the objective function of OPCM to include valuation of shopping time. It should also be noted that these assignment problems are in effect "duals" to the "primal" problem of maximizing satisfaction.

These different values for VALTIM, then, are the values which should be applied in the objective functions above in their respective assignment procedures in the River Basin Model.

With an explicit utility function, the time allocations may be solved for by eliminating the shadow prices from the first-order conditions. Then the values of the shadow prices may be found by substituting the time allocations back into the first-order conditions. If the utility (satisfaction or dissatisfaction) index is given the ordinal interpretation, then only the relative values of the shadow prices can be found uniquely. This is adequate for computing the time valuations needed for the VALTIM.

From the first order conditions it can be seen that the marginal utilities of travel and recreation will be especially important in determining the values of the shadow prices. These determinations bring us to the nature of the objective function itself. A proper discussion of utility functions and their properties cannot be attempted here. With two budget constraints solutions can be tedious, even with such familiar functions as the Cobb-Douglas (which leads to quadratic first-order conditions in the arguments here, on elimination of the shadow prices). A quadratic function would not be too difficult to work with and can be specified to exhibit properties of complementarity as well as substitution along indifference curves.

Attention to a proper role for component $\mathfrak{g}$ is important, for without it education and political action become useless appendages to the model. Since the arguments to $\mathfrak{g}$ --education, politics, cash transfers-- cannot affect satisfaction directly for the current round, $\mathfrak{g}$ must represent a present value of future returns to those allocations. There are several types of returns. One type is the improvement of Pollution, Neighborhood, and Health Index levels in succeeding rounds due to such things as political pressure or cash contributions. These returns are more or less independent of future time allocations. There are other types of returns which will interact with future time allocations, however. Such things as increases in the supply or capacities of public facilities and improvements in the transportation system, again from political pressure or cash contributions, will directly affect the future allocation of time and the levels of the Time

Index. Investments in education can yield returns in terms of better paying jobs, and this may alter time allocations. There may be future cost changes as a result, of course, such as increased tax rates, and these will modify changes in time allocations and thus the present value of the future returns to current allocations.

Suppose, then, that utility (satisfaction) components $\mathcal{U}$ and ϑ are subscripted by round k, $k = 0$ being the present round, and that ϑ_k is the non-time component $\mathcal{M}_k$ of the same round plus the discounted level of the total utility (satisfaction) U_{k+1} of the next round. Then

$$
\begin{aligned}
U_o &= \mathcal{U}_o + \vartheta_o \\
&= \sum_{k=0}^{K} (\mathcal{U}_k + \mathcal{M}_k) d^k
\end{aligned}
$$

where d is a subjective discount factor. $\partial\vartheta_o/\partial t_i$ for education or politics, for example, would be the weighted sum of the partials $(\partial\mathcal{U}_k/\partial t_i + \partial\mathcal{M}_k/\partial t_i)$ where the $\mathcal{U}_k$ are stated in terms of the parameterized forms ofthe optimal time allocations of round k and the parameters of both $\mathcal{U}_k$ and $\mathcal{M}_k$ are written as explicit functions of the t_i of the present round.

To attempt to simulate such an "investment analysis" for population units would not seem feasible, or perhaps even desirable, for this model, so that some proxy or construct will be necessary for $\mathfrak{J}$. Indeed, even with perfect knowledge, finding solutions to such dynamic optimization problems can be quite complicated. However, it might be possible to work out a simple two-period "time preference" analysis which could serve the purpose. Although this will not be done here for the River Basin Model, consider the simple two-period work-leisure-education problem:

Maximize $U = \mathcal{U}(x_k, t_{lk}, d)$

simultaneously over both periods $k = 0, 1$

subject to

$$w_k t_{wk} - x_k - pt_{ek} \geq 0 \qquad \text{(money)}$$

$$T - t_{wk} - t_{lk} - t_{ek} \geq 0 \qquad \text{(time)}$$

for each period $k = 0, 1$

where x_k is consumption, t_{lk} leisure, t_{ek} education time, and t_{wk} is work time, for period k. d is the discount or "time preference" factor, and $w_1 = w_o + rt_{eo}$ gives the second period wage rate as a function of the first period rate and education (assume $t_{el} = 0$). Then if

$$
\begin{aligned}
U &= \mathcal{U}_o + \mathcal{V}_o \\
&= \mathcal{U}_o + d\mathcal{U}_1
\end{aligned}
$$

is the Cobb-Douglas utility function such that

$$e^U = \prod_{k=0}^{1} \left[\left(x_k\right)^{\alpha_x} \left(t_{1k}\right)^{\alpha_1} \right]^{d(\exp(k))}$$

the optimal "investment" in education in the first period is

$$t_{eo} = \mathrm{Max} \left\{ 0, \left(\frac{w_o}{w_o + p} \right) \frac{\left(T - \left(\frac{\alpha_x + \alpha_1}{\alpha_x} \right) \left(\frac{p + w_o}{rd} \right) \right)}{\left(1 + \left(\frac{\alpha_x + \alpha_1}{\alpha_x} \right) \left(\frac{1}{d} \right) \right)} \right\}$$

Analytical solutions such as this could be worked out for the more general problem above with assumptions about the values of parameters for the next round (e.g., "no change"). Of course, the populaton unit allocates for the current round only and is not held to these tentative allocations for the next round.

The Objective Function in the River Basin Model

In the River Basin Model the Time Index is linear in travel, recreation and involuntary time, and no other choice variable enters into the other indices making up the dissatisfaction index. In this case the optimization problem becomes a linear programming problem. The Time Index is the only index directly affected, and it is

$$TIM = t_7 + 5t_6 - gt_5$$

where g is the park utilization factor PRKFAC* .01. If the dissavings limit is not binding then $\overline{V} >$ and $\lambda = \partial\mathcal{U}/\partial\overline{V} = 0$, so that the dollar valuations of time would be infinite. The population unit would want to take the fastest mode or shortest trip (in time) possible regardless of cost. The same applies to cash transfers: if any are made, $\lambda = 0$. These situations are unlikely, so it will be assumed that $\lambda > 0$.

Since shopping and travel both contain some fixed elements, both will be undertaken. Since $-\mathcal{U} =$ TIM we have

$$\Psi_r = \theta + 5$$

$$\Psi_{si} = 5\alpha_i + (1 + \alpha_i)\theta + \beta_i(1 + r_{ai})\lambda, \quad i = 7, 8$$

This leaves five linear conditions in λ, θ and constants, and one in θ and a constant. No more than one need hold as an equation, and it is unlikely that more than two will. This suggests that at most only two of the t_i, $i = 1$ to 5 and $i = 7$, will be positive. Recreation will occur only if it creates less dissatisfaction in shopping and travel than it adds directly in satisfaction, so that shadow prices θ and λ will fulfill

$$\left[g - 5(\alpha_5 + \sum_{i=7}^{8} \alpha_i c_i b_i)\right]$$

$$= \theta\left[(1 + \alpha_5) + \sum_{i=7}^{8} (1 + \alpha_i) c_i b_i\right]$$

$$+ \lambda \left[(1 + r_{a5})\beta_5 + \sum_{i=7}^{8} b_i (p_i + r_i + c_i \beta_i (1 + r_{ai}))\right]$$

Part-time work may be undertaken to support extra recreation[6] or just to help cover fixed costs, and consequently

$$0 = (1 + \alpha_1)\theta + 5\alpha_1 - \lambda\ S_1(1 - r_1) - \beta_1(1 + r_{a1})$$

---

[6] Then $\theta > 0$ and $\lambda > 0$ and with $\partial \mathcal{F}/\partial t_i = 0$ for $i = 2, 3, 4$ there will be no education or political activity.

With both activities pursued there will be a solution for θ and λ from these two equations, and the values for VALTIM(i) can be determined. With no education or politics (see footnote 2), t_1 and t_5 can be solved directly from the budget constraints after substitution for shopping and travel.

There are other possibilities representing the other corner solutions of the linear program, of course. For example, if travel and shopping due to recreation create net dissatisfaction, there may be no recreation _if_ this net is greater than what is created by other activities or involuntary time. There could be part-time work just to eat up involuntary time in such a case, leaving $\overline{V} > 0$ and $\lambda = 0$. On the other hand, if the optimal solution means no recreation or part-time work, then there could be involuntary time (with $\theta < 0$) or politics and/or education (with $\theta < 0$ and/or $\psi_r < 0$). Because this is a linear program these possibilities must be systematically evaluated and narrowed down without the convenience of a simple algebraic determination.

There are some properties of the Time Index which could introduce non-linearities into the problem. Consider the definition of PRKFAC (g above) as

PRKFAC = AMIN1(100, 200 - PRKVAL)

where PRKVAL is the park-use index for the population unit. It happens that PRKVAL is

$$\frac{\text{(other demand at park J)}}{\text{(capacity of park J)}} + \frac{t_5}{\sum_{J'} \text{(capacity of park J')}}$$

when recreation time rather than consumption is used in assignment, J is the park with the lowest PRKVAL,[7] and the other demand specifies use by population units other than this one. Obviously, multiplication of t_5 by PRKFAC in TIM can create a quadratic term. It should be noted that the coefficient of t_5-squared would be positive but very small.

Use of shopping time as proportional to consumption (purchase) but as a function of crowding introduces non-linearities in the cost of shopping, and thus of consumption and recreation, through the proportionality of travel to shopping time. At a facility the shopping time coefficient was defined as

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[7] If PRKVAL is computed as a mean of indices for parks weighted by the population unit's use, the first numerator and denominator will each also be a sum of J'.

$$C_1 = B_1 + B_2 \text{RATIO}$$

where

$$\text{RATIO} = \frac{\text{demand}}{\text{supply}} = \frac{(\text{other demand}) + C_1 x_1}{\text{supply}}$$

Therefore,

$$C_1 = \frac{B_1(\text{supply}) + B_2\ (\text{other demand})}{\text{supply} - B_2 x_1}$$

and $C_1 x_1$, where $x_1 = a_1 + b_1 t_5$, creates a non-linearity in t_5 in the time budget.

A final example is the scaling down of public education time allocation for a population unit by the capacity-demand ratio when demand exceeds capacity. This means that in such a situation if t_2 is the time to be requested for public education, the budget should be charged (and returns based on) an amount

$$\left(\frac{\text{capacity}}{(\text{other demand}) + t_2} \right) t_2$$

This does mean that the sum of reauests could exceed 100 units, and therefore to enforce the budget allocation constraint may force expenditure on involuntary time. However, since the population unit need not actually request time for travel and shopping but should only know their likely values in setting the proper dollar time evaluations, the education request could be scaled upward appropriately. Likewise, recreation time can be over-requested to be safe. This should not be done with other activities under the current priorites of TMALC, since this may preempt time from recreation.

In closing this section it should be reiterated that the "optimality" of this solution is artificial, not only because the dynamism of the model creates uncertainties in the environment and discrete rather than continuous assignments and adjustments, but also because it may not be feasible to allow the population unit to evaluate all opportunities even under limited accuracy in perception. The simplest method is to consider only the locations utilized the past round, so that times and dollar valuations are essentially readjustments for changes in the system the previous round. Reassignment to new locations can still occur, of course. The computational effort could be reduced with a simple non-linear dissatisfaction function, which may also be justified for behavioral reasons,[8] and it is advised that additional variables be introduced to motivate a wider range of behavior. The static model, however, does seem to be an appropriate core of a simulated decision-process.

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[8] Very little attention has been paid in this report to the nature of activity time and money substitutions, chiefly because there exists no real information on them.

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